Re: Catheodory's inequality help please

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From: Isaac (*Isharu_at_yahoo.com*)
Date: 10/15/04

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Date: Fri, 15 Oct 2004 07:43:23 −0400
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"David C. Ullrich" <ullrich@math.okstate.edu> wrote in message
news:217vm0pgor0si3ncamujcr3q5fj3n3d7uq@4ax.com...
> On Thu, 14 Oct 2004 17:37:33 −0400, "Isaac" <Isharu@yahoo.com> wrote:
>
>>How can one prove the following Catheodory inequality :
>>
>>If f is analytic on the closed disk cl(B(0;R)) and M(r) = max {|f(z)| :
>>|z|
>>= r}, A(r) = max {Re f(z) : |z| = r}, then for 0 < r < R, if A(r) >= 0,
>>
>>M(r) <= ([R+r] / [R−r]) * [A(R) + |f(0)|].
>>
>>There is a hint that says consider the case where f(0) = 0 and examine the
>>function g(z) = f(Rz) [2A(R) + f(Rz)]^{−1} for |z| < 1.
>>
>>Well I examined that, and found that |g(z)| <= 1. Also, g(0) = 0, and so
>>therefore we can apply the Schwartz lemma so |g(z)| <= |z|.
>>
>>But what now? I get a new inequality that I don't see what I can do with
>>it. So I cannot see any relation between the hint and what we are trying
>>to
>>prove. Thus how do we prove this inequality? In particular, when you
>>look
>>at this problem, what goes through your mind first, and how do you
>>approach
>>it? I know this is probably a famous inequality, so you might already
>>know
>>how to prove it, but I guess make believe that you didn't know the proof
>>and
>>if you could give some advice as to how you would begin this problem
>>without
>>knowing already how to prove it that would be extremely helpful.
>
> Well you're way ahead of me on this one − I don't see why |g(z)| <= 1.
> (Did you state the problem and the hint correctly?)
>

Yes, stated correctly. Actually, I am wrong about |g(z)| <= 1 (well at least my reasoning was wrong). So I don't see |g(z)| <= 1 anymore. The hint was given as it is stated(I'll note it says "First consider the case ...") so I assume author wants me to deduce something from first considering this case. I guess the problem is that I can't get anything out of this case right now. It looks like a combination of a Schwarz lemma and Maximum modulus principles.

As well, I thought g(0) = 0, but now I don't see that anymore. So I am back to square one. Any ideas?