square through 4 points (was Re Soddy)

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From: philippe 92 (*antispam_at_free.invalid*)
Date: 10/25/04

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Rainer Rosenthal wrote:
> "philippe 92" wrote
>
>>On the Mathworld page about isoperimetric point, I read :
>>(http://mathworld.wolfram.com/IsoperimetricPoint.html)
>>
>>The isoperimetric point exists iff the largest angle of the
>>triangle satisfies :
>>max(A,B,C) < 2 Arcsin(4/5) = 106.26... deg
...
>>
>>Any opinion or point out my mistake ?
>
> Hello Philippe,
>
> do you have any new insights up to now? You are welcome in
> de.sci.mathematik, where some of us like to fiddle around
> with geometry, as you know.

Hello Rainer,

This problem arouse from the Google aptitude test.
Some poster on fr.sci.maths asked for
"how to construct with compass and straightedge a point P so
that perimeters PAB, PBC and PCA are equal".
which *is* question #16 in this test

This triggered discussions about that problem.
I have a rather nice solution with just
4 length copy (= 4 circles)
1 middpoint (= 2 circles and one line)
1 circle
1 perpendicular (= 3 circles and one line)
3 lines

Through my searches on the Web about associated topics came
that angle problem/mistake.

> *These days we discussed a funny question: Think of a square*
> *in the sand. Put stones 1, 2, 3 and 4 on each of the sides*
> *of the square. Wait two days. You see the stones but the*
> *square has vanished in the wind. How to reconstruct it?*
>

That's a completely different topic, and would be worth a
specific thread, as you did in the past about the famous
"maximum triangle" threads.

I've read the thread on de.sci.math, but as my german is
not good enough to post there I didn't interfere.

I knew another solution of that (found in a book), but I admit
Wolfgang's solution is fine !
Your solution is also better than mine. BTW there is a link
between your solution and Wolfgang's.

To enlighten those who read here :
Let PQRS the stones and ABCD the square vertices, P on AB,
Q on BC, R on CD and S on DA.

Wolfgang's solution :
Rotating through the middle M of the square by 90 deg,
PR results into P'R' with P' on BC and R' on AD.
Translating P'R' by vector P'Q results into P"R" = QR"
QR" _|_ PR, and equal.
Hence the construction :
 From Q draw a perpendicular to PR and R" on that
perpendicular with QR" = PR
SR" lies on side AD, draw the perpendiculars from
P and S to SR", and the parallel from Q to SR".

Rainer's :
Draw circles with diameters PQ and PS, intersecting in H (and P)
a perpendicular from P to PH intersects the two circles in
A' and B'. Draw G on PH with PG = A'B'.
GR lies on one side, then construct a parallel from P to GR
intersecting the circles at A and B, and draw the perpendiculars
from A and B to GR.

My comment :
1) These perpendiculars are of course AS and BQ
2) From a remark by Ken Pledger, we get A'B' // RS and equal
So the PG line is the rotated−translated line QS from
Wolfgang's solution. This is the link between the two methods.

I like also Ignacio's fine idea (in Re: Soddy).

Then four constructions up to now :
 − the one I found in a book by J.C Carréga.

On line PS draw H and K with QH _|_ PS and RK_|_PS
On PS draw L with PL = PK − QH
On the perpendicular from L to PS, draw LM with
LM = SH − RK.
PM is on a side. End up with perpendiculars from
Q and S to PM and // from R to PM
This solution is based upon equal angles with
tan(angle) = u/v, and constructing u and v.

- Wolfgang's
- Rainer's
- Ignacio's

Notes, as discussed in de.sci.maths :
1) the problem may be impossible depending on the positions of PQRS
2) if we allow PQRS to lie on the lines AB ... not restricted to
segment AB, Wolfgang said there are generally 6 solutions.

The 6 solutions arise in your method :

- By choosing the two circles
  For a given point R not in these circle, there remains 3 choices
  of circles through P,Q,S : PS+PQ, PS+SQ and PQ+QS
- By choosing G or G' in opposite directions on PH.

This gives 3*2 = 6 possibilities for the side going through R.

Best Regards.

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