Re: Farey Fractions and "reducibility"

Source: http://sci.tech−archive.net/Archive/sci.math/2004−10/7708.html

---

From: Christopher J. Henrich (*chenrich_at_monmouth.com*)
Date: 10/26/04

```
Date: Tue, 26 Oct 2004 22:11:41 GMT
```

In article <1098809467.532406.204860@f14g2000cwb.googlegroups.com>,
rljacobson <rljacobson@gmail.com> wrote:

> The Farey series of order n, denoted F_n, is the set arranged in
> increasing order of all irreducible fractions h/k such that 0<=h<=k<=1
> and gcd(h, k)=1. It is known that if h_0/k_0, h_1/k_1, and h_2/k_2 are
> three successive terms in F_n, then
>
> h_1/k_1 = (h_0 + h_2)/(k_0 + k_2).
>
> However, this is NOT to say that h_1 = (h_0 + h_2) and k_1 = (k_0 +
> k_2), for perhaps one has to reduce the RHS of the above. Indeed, in An
> Introduction to the Theory of Numbers, Hardy and Wright include
> footnotes saying, "Or the reduced form of this fraction", when making
> reference to (h_0 + h_2)/(k_0 + k_2) (p. 23−24).
>
> Suppose h/k and h'/k' are consecutive in F_n, and are separated by
> h''/k'' in F_(n+1). (That is, h''/k'' = (h+h')/(k+k').) Further suppose
> d|(h + h') and d|(k + k') for some d>0. Then d|[k(h+h')+(−h)(k+k')].
> But k(h+h') − h(k+k') = kh' − hk' = 1 where the last step follows from
> the fact that h/k and h'/k' are two consecutive terms in F_n. Hence
> d=1. Therefore (h+h')/(k+k') is in lowest terms and hence h''=(h+h')
> and k''=(k+k').
>
> So why not just state that h_1 = (h_0 + h_2) and k_1 = (k_0 + k_2)?
> Does not this give us more information about the properties of these
> fractions with no extra ink wasted? If the answer is, "Because that
> fact is obvious," then why the strange footnotes in Hardy and Wright?
> (I have not the slightest doubt they knew of this fact.)
>
> Consider this my opportunity to vent some frustrations over a very
> small unnoticed detail setting me back almost a da