Re: Q: properties of covariance matrices

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In article <6b5592.0410272114.5012f641@posting.google.com>,
KY <kky2001@columbia.edu> wrote:
>Let x and y be n−tuplets of normally distributed random variables, and let
> C = cov(x,y') be the nxn covariance matrix of x and y.

I don't think that's what you mean. The covariance matrix of an
n−tuple of random variables X_1, ..., X_n is the n x n matrix
C with entries C_{ij} = Cov(X_i, X_j). For two different
n−tuples, the matrix with entries Cov(X_i, Y_j) is not called
a covariance matrix AFAIK.

A covariance matrix (as I defined it above) is positive semidefinite, and
every positive semidefinite (real) matrix is the covariance matrix of some
set of random variables. With your definition the covariance matrix
would just be any arbitrary n x n real matrix, with no particular
properties.

>Let z be some fixed n−tuplet whose elements are all strictly positive.

> 1) Are all the elements of the n−vector, Cz, non−negative?

No. For example, [use fixed−width font]

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    [ 2 −1 ] [ 3 ] [ 5 ]
C = [−1 2 ], z = [ 1 ], C z = [ −1 ]
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> 2) Is z'Cz > 0?

It's >= 0 because C is positive semidefinite. If C is positive definite
it would be > 0, but if not there may be z for which Cz = 0, e.g.

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    [ 1 −1 ] [ 1 ]
C = [−1 1 ], z = [ 1 ]
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> Where can I read more about bounds on the elements of covariance matrices?

All inequalities on the elements are consequences of the fact that
every principal minor (i.e. the determinant of the matrix formed
by taking some set of rows and the corresponding set of columns)
is nonnegative. Any real symmetric matrix whose principal minors

are all nonnegative is positive semidefinite.

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