

Weak convergence

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Hello,

In the following, \mathbb{C} denotes the complex numbers.

Let E be a complex Banach space, and A a subset of E . My book calls A "weakly closed" whenever for any sequence (a_n) of elements of A , if $\lim x^*(a_n) = x^*(a)$ for any bounded linear functional $x^*: E \rightarrow \mathbb{C}$, then a is in A .

A natural question –that is not mentioned in my book– would be to determine whether a weakly closed set A is closed for the weak topology, whose subbasis is given by $\{x^{*-1}(U); x^*: E \rightarrow \mathbb{C} \text{ bounded, } U \text{ open set of } \mathbb{C}\}$.

I fail when trying to prove that whenever a is in $\text{Adh}(A) - A$, then there exists a sequence (a_n) of elements of A weakly converging to a . The fact is I can control the behaviour of a_n according to a finite number of x^* only. Any suggestion? – a one word answer would suffice, I'd just want to know if I should try proving it or not –.

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