Re: Simple group(s) of order 504?

Source: http://sci.tech−archive.net/Archive/sci.math/2004−10/8633.html

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mareg_at_mimosa.csv.warwick.ac.uk
Date: 10/30/04

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Date: Sat, 30 Oct 2004 10:47:54 +0000 (UTC)
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In article <10o6pkh6dtege78@corp.supernews.com>,
"Jim Heckman" <wnzrfeurpxzna@lnubb.pbz.invalid> writes:
>
>On 19−Oct−2004, mareg@mimosa.csv.warwick.ac.uk ()
>wrote in message <cl3h4eovd1@wisteria.csv.warwick.ac.uk>:
>
>> In article <10n9sm9lt8490f4@corp.supernews.com>,
>> "Jim Heckman" <wnzrfeurpxzna@lnubb.pbz.invalid> writes:
>> >
>> >On 17−Oct−2004, mareg@mimosa.csv.warwick.ac.uk ()
>> >wrote in message <ckth6j$6$1@wisteria.csv.warwick.ac.uk>:
>
>[...]
>
>> >> As for existence, it would not be impossibly difficult to show directly
>> >> that the group <x,w,t> constructed above really does have order 504 and
>> >> is
>> >> simple. I am sure I can come up with a proof of that if you are
>> >> interested.
>> >
>> >Very! Especially if it's "elegant". :−)
>>
>> OK − here goes, but very briefly!
>> We have G = < x,w,t > with
>> x=(2,3)(4,5)(6,7)(8,9), w=(3,4,6,5,8,9,7), t=(1,2)(4,7)(6,9)(5,8).
>
>[snip proof that G is a group of order 504]
>
>> Since G acts 3−transitively, a proper nontrivial normal subgroup could
>> only have order 9 or 72.
>
>Again I'm probably missing something obvious, but how/why does
>3−transitivity limit the normal subgroups?

You can deduce my statement about normal subgroups having order 9 or 72
from the theorem that any nontrivial normal subgroup of a 2−transitive
group is transitive (which itself follows from the more general result
that a nontriival normal subgroup of a primitive group is transitive).

Derek Holt.

>> A normal subgroup of order 9 would be a minimal
>> normal subgroup, and hence elementary abelian, but we know that the
>> Sylow 3−subgroups are cyclic. A normal subgroup of order 72 would contain
>> all 9 Sylow 2−subgroups of G, but then it would have only 9 elements left
>> of order a power of 3, so we would get a normal subgroup of order 9 again.
>> So G is simple.
>
>[...]
>
>−−
>Jim Heckman