# Re: Chances of (random(0,n) + random(0,n) <= m)

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Brendan Sechter wrote:
> *If I am going pick two numbers at random, both from zero to n and add*
> *them, what is the chance of the sum being less than or equal to m?*
>
> *This seems like it should be easy, but my math is really rusty.*
>
> *If n is 5 and m is 3 I can draw this diagram:*
>
> *| m n*
> *| 0 1 2 3 4 5*
> *−−−−−+−−−−−−−−−−−−−−−−−−−−*
> *|*
> *0 | 0 1 2 3 / 4 5*
> *| /*
> *1 | 1 2 3 / 4 5 6*
> *| /*
> *2 | 2 3 / 4 5 6 7*
> *| /*
> *m 3 | 3 / 4 5 6 7 8*
> *| /*
> *4 | 4 5 6 7 8 9*
> *|*
> *n 5 | 5 6 7 8 9 10*
>
> *The chance of the random sum being less than m is 10 / 25. If m is*
> *less than or equal to n, the chances are 0 + 1 + ... m + (m + 1).*

You've got the basic idea.

P(X1 + X2 <= m) = P(X1=0 & X2<=m) + P(X1=1 & X2<=m−1) + ...
+ P(X1=m & X2<=0)

That covers all the possibilities by which these two
numbers can add up to <=m.

If m<=n then the first term is (m+1)/(n+1)^2, the second term is
m/n^2, ... and the last term is 1/(n+1)^2.

That's exactly the sum you came up with, except that
your denominator should be (n+1)^2, not n^2 (there are
n+1 possible values of X1 or X2).

*> If m is greater than n, the chances are n squared minus something. I*
*> can see the "something", but I'm having trouble putting it in terms*
of
*> m and n. I'm also not happy with an if−else answer.*

The above expression is completely general, but some
terms will be zero in the case m>n. That's the "minus
something" you want.

X1 can't exceed n, so all the probabilities X1=n+1,
X1=n+2,... X1=m are 0. There are m−n of these.

(Useful general formula: For a and b integers, a<=b,
there are b−a+1 integers from a to b inclusive.]

Also, the probability that X2 <=m is 1 for m>n.
So the first term is 1/(n+1). The formula
(m+1)/(n+1)^2 should be replaced with
min(n+1, m+1)/(n+1)^2 or (min(n,m)+1)/(n+1)^2

*> Side note: What is my real problem?*
*> This is a death spell calculation that may be used in a game. n is*
*> the spell's power level and m is the target's remaining life. It*
*> would be nice to be able to display the chance of killing the target.*

*> It's not strictly necessary that I understand why a given solution*
*> works, but as a matter of personal pride I'd like to.*

The above should be enough to write a loop that could
do the general calculation in a computer program.

Here's another way to look at it that you might find
useful. Instead of calculating P(X1+X2<=m) directly,
let's look at the distribution P(X1+X2=m)

P(X1+X2 = 0) = 1/(n+1)^2
P(X1+X2 = 1) = P(0+1 or 1+0) = 2/(n+1)^2
(I hope my shorthand is clear)
P(X1+X2 = 2) = P(0+2 or 1+1 or 2+0) = 3/(n+1)^2
...
P(X1+X2 = n) = P(0+n or 1+(n−1) or ... or (n−1)+1 or n+0) =
(n+1)/(n+1)^2
P(X1+X2 = n+1) =
P(1+n or 2+(n−1) or ... or (n−1)+2 or n+1) = n/(n+1)^2
P(X1+X2 = n+2) =
P(2+n or 3+(n−1) or ... or (n−1)+3 or n+2) = (n−1)/(n+1)^2
...

P(X1+X2 = 2n−1) = 2/(n+1)^2
P(X1+X2 = 2n) = 1/(n+1)^2

See the pattern?

The numerators form a triangle shape, increasing from 1
to n+1, then decreasing back to 1 as m goes from 1 to
n to 2n.

The numerator in P(X1+X2 <=m) is thus a triangular number
if m<=n, and the sum of two triangular numbers if m>n.
This is long enough, I think I'll stop here.

− Randy