Re: Skolem's Paradox and why is math the way it is?

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From: J.E. (*troubled6man_at_yahoo.com*)
Date: 11/04/04

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kramsay@aol.com (KRamsay) wrote in message
news:<20041031154428.27108.00001281@mb−m01.aol.com>...
> *In article <39d6e584.0410300900.68138f7d@posting.google.com>,*
> *troubled6man@yahoo.com (J.E.) writes:*
> *|If I use a countable model (or say that V=L) or something, then*
> *|everyone jumps up and down and gets mad at me.*
>
> *I'm not sure what seems angry about the replies.*
>
> *Let me try a different simile.*
>
> *Philosophers sometimes have considered a thought experiment known*
> *as "brains in a vat". They imagine that someone has built a machine*
> *able to exchange nerve impules with brains, so as to make it seem*
> *just like the experience of an ordinary outside world. Then they*
> *consider questions like whether it's possible to know that one is*
> *not inside of such a device, or whether it even makes sense to ask*
> *whether one is (assuming there is no chance of escape).*
>
> *In our discussion of models of ZFC, the models have played a role*
> *something similar to these hypothetical vats. We can imagine someone*
> *arguing that he is not worried about the possibility of being inside*
> *a vat, or whether the things he experiences are really real, as long*
> *as it gives him sufficiently good experiences. There are some who*
> *would claim that as long as the experience was equivalent to what you*
> *would have in a "real world", that there's no meaningful sense in which*
> *they are unreal−− they would say that those things simply *are* the*
> *world for the person inside the box. They might say, for instance,*
> *that it doesn't matter to them whether their spouse is a "real" person,*
> *or a completely convincing simulation, arguing (like Turing) that any*
> *computer capable of generating such a simulation should be regarded as*
> *having the actual mind of the spouse they know. So even if they are*
> *the only brain inside the box (they would argue) they are not really*
> *alone, since they have these AIs to keep them company, ever bit as*
> *"human" as any actual person. These are arguable points, which one*
> *might concede.*

>
> *There's one thing, however, that we can't concede. Suppose that*
> *someone makes a box like this. Then suppose he reasons like this:*
> *since each box is equally real, I can choose to "use" whichever*
> *of these I please as my reality. Hence I can consider the interior*
> *of this box to be my reality. The answer would be: No, you *can't**
> *claim now that the interior of this box is reality. You just built*
> *it! You have these tools here, see? These are *not* part of the*
> *reality projected by the box, however lifelike that reality might*
> *be for someone stuck inside.*

It seems more like a contrived universe where when you try to make a box the cat also gets in the way to ruin it, so you are never sure if one "can" be built, so you don't know whether to take serious the idea that "you" could be inside a vat.

If you look at set "theory" as a list of statements, that say to pick an element (a set) or have your opponent pick an element, pick a path, have your opponent pick a path, until eventually it comes times to ask "does a binary relation b". So you need a function that answers the question "aeb" either yes or no. The FIRST question is, does such function exist that answers "aeb" in such a way that the axioms of set theory hold?

We start out by assuming it does, and then we try to act like the model (the function) is set theory and see what we can "prove" in the model, but the first thing you notice is that the function isn't there, so the idea that the domain or range of the function was the universe was wrong, it doesn't have everything, it never will. So what is the purpose of the set theory axioms. Arthemetic axioms I get, Geometrical axioms I get, Field Axioms I get, Vector space axioms get, but I can't for the life of me figure out what set theory is about.

Yes I *know* that it's about the truth of propositional functions, but the axioms were choosen to model the universe of discourse, and no model is good enough for that, so what's the point? I know that it's PURE mathematics, but maybe I'm old fashioned to think that the point of a theory of propositional relations is so that any theorem of the axiom system is true of any model of the axiom system, and there aren't models of set theory, so it's like we are projectively proving theorems that are supposed to be true for a WIDE class of models, but it seems like we have NO models.

> *There's a corresponding thing in our discussions of models of ZFC.*
> *We have some tools for building models of ZFC. These tools are*
> *themselves mostly set theoretical. So really one is already "inside"*
> *some set theoretic universe already. Some people do this kind of*
> *thing in a universe they regard as actually existing; some merely*
> *pretend for argument's sake that there is a domain of things that*
> *has the properties needed for their arguments to work. They consider*

> it to be a convenient fiction. In any case, we have these tools, and
> you talk about building a countable model using some of them.
>
> In effect, you start out with one notion of set, call it set_1. Then
> in terms of set_1, you define a second, narrower notion, set_2.

We start at the level of having a language. But there are no models of "a theory of all collections of things exententionally possible that don't have too many members", basically because there are things more complicated than the axioms, that could be defined from outside if you had the axioms and a consistent model. The fact that set theory is consistent is NEW information, so if there is a model, it's not a model of "all exetensionall possible sets." There just isn't an intended model of what set theory is SUPPOSED to be. The axioms don't capture set theory. And I've made stronger languages, and it still fails. I tried making stronger axioms, and it still fails. I've tried both and there is just still more existentionally possible sets that aren't in the new models in the stronger langauages and models. I tried going between first and second order logics to see if there was some place to squeeze a model in like Hinitkaa squeezes a truth predicate in it's own language. I couldn't do it. If you go up to second order theory, then there are just more existentionally possible sets, same with third order, I never get Cantor's paradise.

And you can tell me that that is old−hat that other people knew that I taught kiddie set theory to me in school, fine. But then I ask what set theory is a theory OF? What is the point of proving the theorems for a series of unintended models? Group theory is supposed to be about the properties of groups, but set theory has no "sets" because any candidate is too small to be all.

> Now, just as with the brain−box, it doesn't make sense to say, after
> having build such a model, that you now elect to regard it as being
> the whole set−theoretical universe. Why? Because here you are with a
> wrench in your hand, which is demonstrably outside of the box. If you
> are inclined to "care" only about set_2s, you're sort of stuck, because
> you defined them in terms of set_1s.

Since there are no intended models, how do we know that we aren't in an unintended model and that bigger tools that we can't see SHOULD be defined, isn't that what we DO with second order logic, you postulate the existance of tools for set theory, but then you just get more sets, and there should be tools for creating them. It's like DnD, where if you have armor−making tools, and armor−making skill, then you can make armor, but heaven forbid the player askes who makes the armor−making tools. Someone could have armor−making−tools−making tools and armor−making−tools−making skill, and then make armor−making tools, but HE needs a guy with the skill and tools to make his tools. I thought this was silly when I heard the rules, but set theory seems to be the same way, the standard interpretation is to have every exestentionally possible set, but the tools are always inadequate to get them all, no matter what tools you have.

> *I don't find it clear that you really have accepted the impossibility*
> *of defining a countable model, and then considering it to be the whole*
> *set−theoretical universe. Maybe it's just something that gives you an*
> *impression of being peculiar. It's still not clear what your point of*
> *view really is. But you keep saying things that sound as though your*
> *decision to "use" a countable model should be able to work some other*
> *way than FIRST accepting some larger domain of sets, THEN deciding to*
> *consider a subdomain inside it.*

It doesn't seem harder than considering ANYTHING ELSE to be the whole set−theoretical universe, they are ALL bad models. But the countable one is that gets the criticism as not being good enough for physics.
I still don't get that. The countable model can do everything ZFC can do, so what if it doesn't do everything that ZFC+CON(ZFC) can do? To say that the countable model is NOT good enough for physics is the same as saying that ZFC is NOT good enough for physics. Sure the universe of ZFC+CON(ZFC) is bigger and "better" mathematically than the universe of ZFC, but why is this bad for physics, why the dire predictions about not "having limit points" for sequences outside the model INSIDE the model? So we don't have every exestensionally possible set, neither do YOU, so why the holier than thou? Do you understand my confusion yet?

This gets back to the "why" of set theory. If "not having every exestentially possible set" is grounds for saying a model is bad, then WHY prove theorems about set theory since EVERY model of set theory is lacking existentionally possible sets? Physics needs to model every physically possible thing, can't we find some framework big enough for that, why try to chase an ephemeral theory that has no models. In physics we might find a TOE, but matheamtics isn't going to, so do we need a math TOE to find a phyics TOE, it seems like an onerous burden, who's necessity escapes my comprehension so far.

> *The only way to keep it from being circular is to base your acceptance*
> *of the first domain of sets on something not requiring any set theory.*
> *Now you can go either down the realist road, treating them like any other*
> *entities of which we have theories, or down the nonrealist road, where*
> *you explain them away as merely hypothetical or fictional. But you can't*
> *start with a model that depends for its definition on an already existing*
> *set theoretical domain.*

All other axiom systems in mathematics are about proving a theorem of an axiom system so that it's a truth in all models. This is supposed to be the POWER of mathematics, that a group is a group is a group, no matter where or how it is found. But there are NO models of set theory. So we aren't proving anything, we made axiom systems so broad that no models exist. What was/is the point? What is it about? If these axioms have no models, why these axioms?

> |*But "to me" that's*
> |*what the mini−people are doing, and I don't see minipeople on the*
> |*minidock jumping up and down at some mini−J.E., so I go to usenet to*
> |*ask what *I'm* doing wrong that no one else does. If using a*
> |*countable model is "wrong" for me (even though my "universe" could be*
> |*a countable model to a super−large person), then I should be able to*
> |*see it being "wrong" for the mini−people. All I see is that either no*
> |*one is wrong or everyone is wrong, because it's all the same formal*
> |*system everytime.*
>
> *This seems like a place where the metaphor really starts to break down.*
>
> *To strain the metaphor a bit, though, what is wrong is to pretend one*
> *both can and cannot describe the bubble around the dock from some*
> *outside vantage point. If you can, then you can refer to the structures*
> *lying outside of it, which means that the people inside the bubble can*
> *say, "the reals we can see may be countable, but if so then there are*
> *reals that we can't see". If you cannot describe the bubble from an outside*
> *vantage point, then it makes no sense to even consider the possibility*
> *of it's turning out "actually" to be a bubble inside of a larger dock.*
>
> *A book by Ronald Dworkin called _The Law's Empire_ has some interesting*
> *comments in the first chapter or two about different kinds of skepticism.*
> *One relevant issue is distinguishing between a kind of skepticism that*
> *assumes the validity of a framework, and the kind that denies the validity*
> *of the framework.*
>
> *Keith Ramsay*

I might agree that methaphorical reasoning may be reaching its limits. Am I wrong to think that the people on the outside have stronger tools, that the people on the inside could, by adding axioms create stronger tools than they currently posses, and that we would "approve" of these tool choices in a way that we would "not approve" of GCH or something being taken as a new axioms, because these tools create truths that SHOULD be true, as opposed to GCH which creates truths that are opposing ~GCH, which is JUST as valid. But then this implies that we should be making better tools ourselves, because "brains in a vat" our axioms could be a small model in someone else's model.

Either ZFC is BAD and we need new axioms, or ZFC is FINE (for physics) in which case a countable model is OK (for physics). I think it is my realist opponents that want it both ways, not me.