Re: Counterexample to t((c^n − a^n) mod b) | phi(b)

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Doug Goncz <dgoncz@aol.com> wrote:

> "If s,y,z are non−zero integers such that x^n + y^n = z^n, if d=gcd(x,y,z)
> and x1=x/d, y1=y/d, z1=z/d then x1^n+y1^n=z1^n, (which I totally get)
> where the non−zero integers x1, y1, z1 are pairwise relatively prime
> (which I don't get). So if we assume that Fermat's equation has a
> non−trivial solution, it has one with pairwise relatively prime integers."
>
*> I just don't see how factoring out a denominator common to all *three* of*
> x,y, and z leaves x and y, y and z, and x and z with no common
*> denominator, in *pairs*.*
>
> n,a,b,c = 2,3,4,5 or 2,6,8,10 are solutions to a^n + b^n = c^n. And I've
> never seen a (base?) Pythagorean triangle that had a common factor between
> two edges.
>
> It seems to me x=x1md, y=y1md, z=z1d is possible.

In the particular case of x^n + y^n = z^n, any prime which factors two
of x, y, and z must also factor the third.

For example, suppose p | x and p | y.
Then p | x^n and p | y^n.
Then p | x^n + y^n = z^n.
And one of the properties of prime numbers lets us use p | z^n to
conclude p | z.

The same works for subtraction as well as addition, in case z is one of
the two.

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