Re: tetrahedron problem

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Robert Israel wrote:

> A and B are in S with d(A,B) = sqrt(7/2 − sqrt(6)) > 1.0249.
> I believe this is the maximum possible distance between points
> of S.

Refer to Clive Tooth's animation for a picture of A and B. The arced edges of the figure are segments of circles formed by the intersection of the two spheres of radius one with centers on the vertexes of the opposite edge.

They have radius sqrt(3)/2 and centers at the midpoints of the opposite straight tetrahedral edges. The radius is the altitude of a face bisected by the plane of one of the circles.

Since opposite edges form the diagonals of opposite faces of a cube, the distance between midpoints of opposite straight edges is 1/sqrt(2) .

This means that the length AB is 2sqrt(3)/2 − 1/ sqrt(2) = sqrt(3) − 1/sqrt(2) which agrees with Robert Israel's result.

One can make a qualitative argument that the distance AB is greater than 1, for consider the circle of radius 1 through A or B. It passes through the vertices of the opposite edge since it lies on the spheres of radius 1 having those vertices as centers. But the arced edge passing through those vertices has radius less than one, and so lies outside the circle of radius 1 just described, and all its points have distance >1 from the center of that circle.

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