Re: Cantor's diagonal proof wrong?

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curt@kcwc.com (Curt Welch) wrote in message news:<20041114013915.877$0a@newsreader.com>...
> *Here's something all of you should have some fun with.*
>
> *Nath is not something I specialize in (and I don't read this group*
> *normally), but I've been looking at a few things lately and I've decided*
> *that some very big mistakes have been made in math because people started*
> *playing around the concept of infinity without realizing the trouble they*
> *were creating for themselves.*
>
> *When I was shown Cantor's diagonal proof that the number of reals was not*
> *countable back in college, I thought it was a fascinating proof. It seemed*
> *to uncover some great mystery about the nature of numbers that was not at*
> *first obvious. It sounded very logical and I quickly embraced it as fact.*
>
> *Lately however, I've come to see things very differently. I now belief the*
> *proof is totally bogus. And the huge body of work built on top of the*
> *concept is likewise, totally bogus.*
>
> *But, like I said, I'm not a math expert by any means. So I'm posting the*
> *idea here so you experts can have fun laughing at me.*
>
> *The reason I came to these conclusions is because I've spent a lot of time*
> *trying to uncover the mysteries of AI (artificial intelligence). i.e.,*
> *trying to understand how to build a machine that is just as intelligent as*
> *we are. And as I've worked on that, I've come to some understanding (also*
> *unproven) about what "thinking" is all about. And this led me to question*
> *some fundamental ideas in other fields, like math.*
>
> *The problem with math, is that when you start playing with ideas like*
> *infinity, you are making some basic assumptions on what an idea is. Yet,*
> *no one knows how we think, or why we think. Yet, that hasn't stopped*
> *mathematicians and philosophers in making endless assumptions in order to*
> *try and understand the very nature of thought (and existence), and how it*
> *applies to their field. However, I think they got a few things very wrong.*
> *And it's my work on AI which caused me to question Cantor's diagonal proof*
> *in the first place. And instead of accepting it as fact, as I did back in*
> *college when it was shown to me, I looked at it again thinking it was*
> *invalid. And when I look at like that, I see that it is.*

>
> *Let me demonstrate.*
>
> *I claim that there is only one type of infinity. That there are just as*
> *many integers as there real numbers. (or more accurately, that the concept*
> *of the size of an infinite set is a contradiction in itself).*
>
> *So, let me create a mapping. I'll start with the mapping from the integers*
> *to the reals in the range 0 to 0.99999....*
>
> *I R*
>
> *0 0.0000...*
> *1 0.1000...*
> *2 0.2000...*
>
> *10 0.0100...*
>
> *123 0.3210...*
>
> *So you just reverse the digits in the integer to create the real. I claim*
> *this mapping is one to one and covers all the reals in that range. For any*
> *real you give me, I can easily give you the matching integer.*

****
****

Curt, This should be an easy one for you, then. I give you the (irrational) real 0.1 2 3 4 5 6 7 8 9 10 11 12 13 14... which is constructed by concatanating all the integers after the decimal point, ad infinitem. I firstly appeal to you to recognise that the length of that (irrational) real is the kind of infinity you said you are familiar with and which you accept. Secondly I appeal to you to recognise that this is indeed an irrational, the recipe (algorithm) of its construction ensures there is no repetition anywhere after the decimal point (this can be proven rigorously but I don't think we need to bother). Being an irrational I would now kindly ask you to write this real down in your reverse fashion. What is the first digit?
Another example, there are algorithms for the construction of Pi = 3.14159... ad infinitum, non repeating, truly irrational, but cannot be as easily proven as the number stated above. Your scheme presupposes there is a last digit in Pi, how else are you going to write it down?. You say all you need is the algorithm that constructs the real and you use it to right down the matching natural. What is the last digit produced in a non−ending process? because this is what most algorithms that produce reals are. Not the algorithms is infinite, it can usually written down in a couple of lines, but the proces is which it describes and which spits out the digits.
But even if you hadn't complicated matters for yourself unnecesssariy by inverting the sequence of digits, it makes no difference: what you are doing is writing down a list of terminating sequences, all of which are, of course, reals of the rational kind. And the number of

rationals is of the kind of infinity you accept, the same infinity as the natural numbers.
The majority of reals, even the majority of the rational kind of reals, are non−terminating, in other words, infinite sequences of digits after the decimal point. And by contrast, every natural number has only a finite number of digits! That number may be arbitrarily large, but it is a finite number.
Your 'writing the digits back to front' just gave me a thought. The never ending sequence of digits of Pi = 3.14159... produced by one of the many algorithms for Pi is a number. 0.123456789101112131415... ad infinitem in the example above is a number.
...951413.123456789101112131415... where the sequence to the left of the decimal point is computed by the algorithm that produces Pi, if the ... to left indicate a never ending sequence, is NOT a number.
Like you I have never been able to come to terms with 'infinity' or the various kinds of 'infinities' as numbers, the kind of stuff that eventually drove Georg Cantor, their inventor, to insanity. At one stage I grasped 'infinity' as a serious of steps, which I could choose to let never end (as a mind game), and I have since been perfectly at ease with Peano's axioms and inductive reasoning, and all the good things that flow from it for the working amateur or mathematician, but I have never even accepted the size of the set of natural numbers, 'aleph' as a number. aleph + 1 is totally meaningless to me, as is 2 raised to the power of aleph. Unless I can interpret these constructs somehow as processes for counting, and in this regard give me something I haven't already got with the naturals, I have no use for them. Starting from the basis of the naturals we construct the integers and the rationals. Don't you find it exciting to define an order in the naturals and rationals, then realise that given any two rationals a and b, no matter how close (a+b)/2 lies between them, and that this gives rise to the realisation that the rationals are everywhere dense. Seemingly not a hole in sight, if there is one we quickly plug in as many rationals as we need to fill it [by using (a+b)/2]. And along comes this pre−Euclidean renegade and points out that if we use multiplication in novel ways and not only ask what do we get if we multiply a*b, but what number if multiplied with itself will produce a given number, he points out that a number x, multiplied with itself to give 2, is NOT rational. One of the most famous proofs at all, and passed on by Euclid. The renegade enraged his Pythagorean mates so much that he condemmed to die, and drowned somewhere in the Mediterranean. So now we have hole in the presumably dense rationals that we cannot fill. It took many years of sweat and tears until order was restored. But we don't want to go into that, all I want to say is that the rationals can be mapped 1−1 with the naturals, huge subsets of the irrationals even can be mapped 1−1 with the naturals, for example the algebraic numbers which arise from polynomial equations, of which the square root of 2 is but one example, being the solution of the polynomial x^2=2. But above that is the towering infinite mass of all the other irrationals, the so−called transcendental number. Of the different equivalent ways of constructing the reals then, for example as sets of rationals (so called Dedekind cuts), their

construction as decimal expansion is the one used in Cantor's diagonal argument. I cannot find any flaw in Cantor's diagonal proof, but admit that every once in a while I have to really concentrate to answer the following question: let us have an infinite list of decimal expansions, but let each of them be a rational number. This could mean a decimal expansion non−repeating out to billion^billion−th place, then have a non−repeating sequence of decimals stretching as far again, which then repeats into infinity. Imagining this infinite list of rationals I set myself the task of NOT using Cantor's diagonal proof to show that it necessarily produces a decimal expansion not already in that list.

I hope you find my argument convincing.

By the way, I hold (with many others) that mathematics is what amateurs and mathematicians do. There is no use frying your brain over the question whether mathematical objects exist, or if the exist, what they are. Admittedly, mathematics has been incredibly successful in describing whatever else man tackled in the sciences in the last 350 years, but it is best to accept that mathematics is what mathematicians do, else doing math stops being fun and you soon find yourself staring into the abyss.

Cheers, Reinhard
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Just reverse
> *the digits. This includes all the irrationals because in fact, you can't*
> *give me an infinite irrational to work with. You can only give me an*
> *algorthm for creating it. And any algorithm you give me, I can modify to*
> *create the matching integer.*
>
> *Let's look at this mapping with Cantor's diagonal proof. We construct a*
> *real number by picking digits from the diagonal which is different from*
> *each row in the table. Well, as it happens, the diagonal in this mapping*
> *is all zeros, so we can pick a simple real like 0.1111... as the number*
> *which can not be in the table. I'll call this number D. Cantor's proof*
> *seems to show quite clearly that D is not in the table, because it can not*
> *be located at any row of the table (for what seems to be obvious reasons).*
>
> *Let me define D(n), as the first N digits of this "constructed" missing*
> *real diagonal. The first N digits are 1, and the rest are 0.*
>
> *So D(2) is 0.11 and D(5) is 0.11111 etc.*
>
> *We see that D(5) can not be located in the first 5 rows of the mapping.*
> *But, we also can easily prove that D(5) does show up at row 11111.*
>
> *So, as we construct D(n), we see that even though it doesn't match any of*
> *the rows up to the point we have reached, it is always further down in the*
> *table. And because the table is infinite, we will always be able to find*

> it futher down in the table.
>
> So, this proves that D(n) for all values of n, from 0 to infinity, is in
> the table.
>
> So, now we have a contradiction. Cantor's proof says that D(infinity) is
> not in the table, yet D(n) for all values of n, including infinity, is in
> the table from the above logic, which is just as clear and straight forward
> as Cantor's. So how can both be true? If they are not both true, which is
> one wrong and the other one not?
>
> How can this be? I say, it's because there's a contradiction in Cantor's
> proof, and the contradiction is not the one that everyone assumes − that
> there are more reals than integers.
>
> As another example, let me show that the number of integers are also
> greater than the number of integers, using the logic of Cantor's proof.
>
> Lets create a table of integers like this:
>
> ...000000
> ...000001
> ...000002
>
> ...000010
>
> ...000123
>
> It's just a normal list of integers, but instead of following the normal
> convention of leaving off the leading zeros (which we all know are implied
> even if we don't write them) I include them in that table.
>
> So lets use Cantor's logic on this table and see if we can construct a
> number which is not in the table. We take the numbers from the diagonal,
> and construct the number ...111111 just like we did above.
>
> Since we construct this number by changing a digit from every row, we know,
> by Cantor's logic, that the resulting number can not be in the table.
> Therefore, with the wisdom of Cantor, I've proved that the number of
> integers is greater than the number of integers. There are some integers
> which are simply not in the list of all integers.
>
> Ok, so if Cantor was wrong, why was he wrong?
>
> The answer is one already well known to mathematicians. They just never
> realized how it applied here. You can't use infinity as if it existed. It
> doesn't exist. "infinity" is only a name for something which can not
> exist.
>
> The contradiction that Cantor put into his assumptions in the diagonal
> proof, was that something of infinite size does exist. The number I call

> D, the constructed diagonal which can not exist, he declares does it exist.
> That's the contradiction in his assumption which causes the final
> contradiction.
>
> If you think it's ok to use infinity like it was real, it becomes possible
> to prove anything by contradiction. I can easily for example prove that 1
> = 0 by making the same mistake by playing with an infinit series of 1 − 1 +
> 1 − 1 + 1 ..., or by using 1/0 in a proof as if it were a number that
> existed.
>
> So, what I'm saying is that infinite sized sets don't exist at all, and
> can't exist. And any time you start with an axiom which says "infinite
> sized sets do exist", you have introduced an contradiction into your axioms
> which guaranties contradictions in your results.
>
> We don't have "the set of all integers". What we have is a counting
> algorithm that can generate as many integers as you need for any
> application.
>
> It's perfectly valid to talk about what infinite algorithms do as they
> approach infinity. But once you start to pretend they reach it, you have
> stepped over the line into a world filled with contradiction, and a world
> which has nothing to do with the universe we exist in.
>
> This is because "ideas" are not "magic". They are the result of mechanical
> computation. And mechanical computation takes time. So any time you talk
> about computing an infinite sized set (like the set of all integers) you
> have stepped outside the realm of reality and into a fantasy world full of
> contradictions which you created by putting the contradiction of the
> existence of infinity into your world. If you start an algorithm running
> to create your infinite sized set, it will never finish, so any attempt to
> talk about what you do after that is invalid. In Cantor's proof, he asked
> us to construct an infinite sized real, and then check to see if it was in
> one of the rows of the table. And as I showed above, as you construct it,
> the number you have will always be in the table. No matter how long you
> spend constructing D, the value you have will always be in the table.
>
> If you "pretend" the job of construction does end (the program that can
> never halt does halt), then you have put a contradiction into the system
> that allows you to prove anything by contradiction. You can prove there
> are more reals than integers, or that there are more integers than integers
> (as I did above), or that 1 = 0. As long as you can slip that
> contradiction in as an axiom (without anyone raising a penalty flag), you
> can prove anything you want by contradiction − because you started with a
> contradiction.
>
> Much other important work, such as Gödel's, also fell prey to this same
> mistake.
>
> Oh, and if you want a mapping from the integers to all the reals, here's
> one:

>
> 0 0.0
> 1 0.1
> 10 1.0
> 123 2.31
>
> i.e., you take the integer and number the digits like this:
>
> ... D4 D3 D2 D1 D0
>
> And you construct the real as: ... D3 D1 . D0 D2 D4 ...
>
> You use the even digits to construct the real to the right of the decimal
> point, and the odd digits to construct the real to the left of the decimal
> point. So your integer which grew to infinity in one direction, now
> creates a real which grows to infinity in two directions.
>
> Now, I know most (if not all of you), will tell me I'm crazy. Many
> probably won't even read my post. But if you think I'm crazy, tell me
> this. How is my proof that the number of integers is greater than the
> number of integers, any less valid than Cantor's proof that the number of
> reals is greater than the number of integers? If you can't tell me that,
> then why would you believe Cantor's proof is valid? If you can, I'd like
> to hear about it.
>
> Has any one else put forth this same argument (or others) that Cantor's
> proof is invalid?