Re: Prime ideals in Z[x]

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Arturo Magidin wrote:
>
> *No, the Krull dimension is no guarantee; you have there an example of*
> *a UFD which has Krull dimension greater than 1; and Z[sqrt(−5)] is an*
> *example where the Krull dimension is 1 but the ring is not a UFD.*
>
> *I'm not entirely sure what he meant; you can get arbitrarily high*
> *Krull dimension and still have a UFD, simply by taking things like*
> *Z[x1,....,xn]. Then you have the chain*
> *0< (x1) < (x1,x2) < (x1,x2,x3) < ... < (x1,...,xn) < (2,x1,...,xn)*
>
> *so the dimension is at least n+1. On the other hand, "all but one" of*
> *those prime ideals come from the "transcendence degree".*
>

More generally, a given integral domain R is a UFD if and only if all prime ideals of height 1 are principal. It is easy to show that any prime ideal of height r>0 must be generated by at least r elements. The above tells us is that the converse holds for r=1 <=> R is a UFD.