Re: Symbolic solution of quadratic matrix equations

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From: Gregg Kelly (*greggk_at_ozemail.com.au*)
Date: 11/30/04

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In answer to Dave's question below, I believe I can shed a little more light on this. Rewrite the matrix equation as

A.X^2 + B.X + C = 0 (1)

Let k and v be an eigenvalue and eigenvector of X so X.v = k.v
Then multiplying (1) by v gives

(A.k^2 + B.k + C).v = 0 (2)

so that

det(A.k^2 + B.k + C) = 0 (3)

This is then a polynomial equation of degree 2.n for the n eigenvalues of X. Now we take n solutions k_i of (3) we can then solve (2) for v_i.

Then X is given by the eigen decomposition formula

X = V.K.V^−1

where

K = diagonal matrix k_i
V = matrix of vectors v_i

Which n solutions of (3) should be chosen I don't know. Maybe they all give valid solutions, maybe only some of them do ...

rusin@vesuvius.math.niu.edu (Dave Rusin) wrote in message news:<cog70j$l5u$1@news.math.niu.edu>...
> In article <codc2i$2p5$1@nntp.itservices.ubc.ca>,
> Robert Israel <israel@math.ubc.ca> wrote:
> >In article <b73a68dc.0411280909.64d5dbbe@posting.google.com>,
> >Ceylan <Ceylan.Yozgatligil@gmail.com> wrote:
> >

> >>I need to find a symbolic solution of a quadratic matrix equation
> >> X^2 * A + X * B + A' = 0
> >>where A' is the transpose of (kxk) nonsingular matrix A and B is a
> >>(kxk) symmetric matrix.
>
> >This is not going to be easy, except in the case
> >where all the matrices commute. You can treat it
> >as a system of k^2 quadratic polynomials in the entries
> >of X, and use try Groebner basis techniques, but I'd guess
> >it's likely to be rather complicated unless k is very small.
> >A 2 x 2 example should work pretty well, but even 3 x 3
> >might be very ugly.
>
> Of course Robert is correct; this is going to be a mess.
> But it looks like there IS some structure to the answer;
> I don't have an explanation for it but I offer my findings