Re: Uncountable many reals without Cantor

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From: Jürgen R. (*jurgenr_at_web.de*)
Date: 12/01/04

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Date: Wed, 01 Dec 2004 09:31:27 GMT
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David C. Ullrich <ullrich@math.okstate.edu> wrote:

>On Tue, 30 Nov 2004 08:09:18 GMT, jurgenr@web.de (Jürgen R.) wrote:
>
>>David C. Ullrich <ullrich@math.okstate.edu> wrote:
>>
>>>On Mon, 29 Nov 2004 12:15:52 +0100, Frank Piron <empty@zero.nil>
>>>wrote:
>>>
>>>>Hi,
>>>>there are often threads in this group concerning
>>>>the cardinality of the set of real numbers. Some
>>>>persons seem to have strong objections against the
>>>>Cantor Proof of the fact that the set of real
>>>>numbers is not denumerable by the naturals.
>>>
>>>People may have "strong" objections, but nobody
>>>has any _coherent_ objections − the people who
>>>object seem to be unable to follow very simple
>>>reasoning. Hence I doubt that they're going to
>>>be able to follow complicated chains of reasoning...
>>>
>>>>Cantor's Proof uses diagonalization. But there is
>>>>a mesaure theoretic argument for the uncountability
>>>>of the reals due to Borel which does not use this
>>>>technique.
>>>>
>>>>Let (a_i), i e {1,2,3,...} be a list of the reals in
>>>>the interval [0,1]. Let eps be any rational number
>>>>> 0.
>>>>
>>>>Now consider a_1 in an interval of length eps/2, ...,
>>>>a_i in an interval of length eps/2^i. Since every
>>>>element of [0,1] is in some of the intervals, we
>>>>have
>>>>
>>>>length([0,1]) <= eps/2 + eps/4 + ... + eps/2^i + ... = eps
>>>>

>>>>for every rational eps > 0. A contradiction.
>>>
>>>I can imagine one of the objectors mentioned above
>>>_agreeing_ that this argument is right, because it's
>>>based on more familiar concepts. But I think the idea
>>>that it's actually simpler is bogus − if someone
>>>agrees to this but not to the diagonal argument I
>>>really don't think that he's understood all the details.
>>>
>>>This argument _is_ much more complicated, if you include
>>>the missing details. In particular you need a _proof_ of
>>>the intuitively reasonable fact that if [0,1] is contained
>>>in the union of countably many intervals I_n then
>>>
>>>(*) sum length(I_n) >= 1.
>>>
>>>How do you _prove_ that?
>>
>>Assume the opposite, put the intervals end to end etc. This kind of
>>thing is proven in the beginning of any Real Variables text, e.g.
>>Royden.
>
>Well of course this is proved in reals (although it's not so
>clear to me that "put the intervals end to end" has much to
>do with a proof − never mind that, not really relevant.)
>
>>Where do you see a problem?
>
>I don't see any problems with the validity of the proof.
>I question the relevance in the present context because
>when all the details are included it's much more complicated
>than the diagonal argument.

OK, agreed.