

Re: The origin of quaternions – Hamilton's 1844 paper

Source: <http://sci.tech–archive.net/Archive/sci.math/2004–12/3704.html>

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Date: 12/01/04

Date: Wed, 01 Dec 2004 17:00:31 +0100

Perhaps the following is a relevant contribution to the newsgroup thread on the origin of quaternions.

W.R.Hamilton's second paper on quaternions presented to the Royal Irish Academy:

W.R.Hamilton: On quaternions, or on a new system of imaginaries in algebra; with some geometrical illustrations.

Communicated November 11th 1844 to the Royal Irish Academy, Proc. Roy. Irish Acad. vol. 3 (1847), 1–16.

If I remember well, it is in this paper that Hamilton tries to set up an algebra of proportionalities of spatial directions.

In the horizontal plane one can say: North is to East as South is to West. In the meridional plane one can likewise say that Zenith is to South as Nadir is to North. These are geometrical rephrasings of properties of the complex number system.

Hamilton did not succeed in obtaining a consistent algebra of spatial directions. Instead he found that when one more dimension to the three dimensions North–South, East–West, Zenith–Nadir, one indeed obtains a nice closed and consistent algebra of directions and their quotients.

All this is actually a geometrical rephrasing of Hamilton's discovery (or invention?) of the quaternion algebra.

H. knew that this fourth dimension can in no way be a spatial dimension.

He imagined this dimension as "laid out on a scale" and coined the term "scalar" for this kind of numbers. H. was predominantly a physicist and so identified this fourth dimension with time. To no avail, as would become apparent over the decades to come.

In hindsight one may guess that Hamilton's motives for his quest were (1) to generalise the algebra of complex numbers to 3D, and to maintain in the process: (2) the concepts of quotient of directions and of quotient of vectors, and (3) the law of moduli of complex numbers, and of course (4) the often repeated questions at breakfast by his sons, aged six and eight in the autumn of 1843: "Well, Papa, can you multiply triplets?" (*)

All this still does not answer the questions of exactly how H. got his brainwave to add a fourth dimension, and of why he got it then and there. My cherished speculation is that Hamilton's walk on October 16th 1843 from Dunsink to the city of Dublin helped to clear up his mind and open it up for a brainwave (a flow of endorphin set in motion by enjoying a 5-mile walk in the cool autumn air). And then the brainwave will of course involve the subject closest at hand.

(*) Seán O'Donnell: William Rowan Hamilton – Portrait of a Prodigy. Boole Press Dublin, 1983, ISBN 0-906783-06-2 (hc) and 0-906783-15-1 (pbk)

Johan E. Mebius

Ken Pledger wrote:

>In article <41AB6DE9.10909@earthlink.net>,
> Roger Bagula <tfn@earthlink.net> wrote:
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>>My understanding was that he was studying vector analysis
>> and looking at A divid B for two vectors.
>>(ref. page 15 Theoretical Mexhanics, Ames and Murnaghan, Dover,1957).
>>" The consideration of A divid B , not as a vector but as an operation
>>carrying a representative segment B into
>>a coterminous representative segment of A , led Sir William Hamilton to
>>the study of quaternions."....
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> This may be a misunderstanding. I remember that one of Hamilton's
>text-books does begin with the idea of dividing one vector by another,
>but that was written long after the original discovery/invention.
>
> Ken Pledger.
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