

Re: trig identity help

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tsmith wrote:

> *Hi,*

>

> *In my physics book, for superposition of waves it uses the identity*

>

> $\sin(a) + \sin(b) = 2\cos((a-b)/2)\sin((a+b)/2)$

>

> *I looked in my calculus book's trig review appendix and they do not prove*
> *this one.*

>

> *So how to derive it?*

$\sin(A+B) = \sin A \cos B + \cos A \sin B$ (1)

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

Add:

$\sin(A+B) + \sin(A-B) = 2 \sin A \cos B$

Put $A+B = a$, $A-B = b$

then $A = (a+b)/2$, $B = (a-b)/2$

Hence

$\sin a + \sin b = 2 \sin((a+b)/2) \cos((a-b)/2)$

To derive (1) use

$\exp(ix) = \cos x + i \sin x$

so

$\sin x = \text{Im} \exp(ix)$ (for real x)

Thus for real x, y

$\sin(x+y) = \text{Im}(\exp(ix)\exp(iy))$
 $= \text{Im}((\cos x + i \sin x)(\cos y + i \sin y))$
 $= \cos x \sin y + \sin x \cos y$