Re: .99999... still=/= 1

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From: Dave Seaman (*dseaman_at_no.such.host*)
Date: 12/21/04

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Date: Tue, 21 Dec 2004 17:00:21 +0000 (UTC)
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On Tue, 21 Dec 2004 15:45:34 GMT, Tapio wrote:

> "Dave Seaman" <dseaman@no.such.host> wrote in message
> news:cq7ohuvlf1@mailhub227.itcs.purdue.edu...
>> On Mon, 20 Dec 2004 20:46:26 GMT, Tapio wrote:

>>> "Dave Seaman" <dseaman@no.such.host> wrote in message
>>> news:cq4d8t$4vm$1@mailhub227.itcs.purdue.edu...

> (snip)

>>>> but there is no such thing as a
>>>> smallest infinite hypernatural. If n is an infinitely large integer,
>>>> then so is n−1.

>>> Omega was defined in this discussion earlier as follows:
>>> The number that is greater than any infinite integer. That is the
>>> smallest
>>> transinfinite number. Omega is the successor of ...999, i.e. n+1. The
>>> precessor (n−1) of ...999 is ...998.

*>> It's possible to define the hyperordinals and to talk about *omega in*
*>> NSA, but this *omega has no relation to anything you said in that*
*>> paragraph. For one thing, *omega is not a member of *N. (In fact,*
*>> *omega is identical to the set *N itself).*

> Correct, because that omega above (maybe some other name is better − let's
*> use your symbol *w ?) is in the next infinite area. The reasons are:*
> Every number has a successor, i.e. you can always add 1. All the
*> placeholders in the infinite long area *N were already occupied.*

I don't know what you mean by an "infinite area" or by a "placeholder".

>> For another, there is no
>> connection between the hyperordinals and strings (even hyperstrings!) of
>> decimal digits. I don't accept your definition.

> I could not follow you above, sorry. :−(

>> Are you under the impression that ...999 is a hyperinteger? It isn't,
>> unless you explain which equivalence class of sequences of integers you
>> are talking about. I can think of ways to make such a correspondence,
>> but you haven't said what you mean.

> OK, let's try again. First of all, I think we should start a new thread from
> the beginning so that everything is constructed and we use the same concepts
> and definitions.
> 1) ...999 is not N as it is not in a classic way finite integer. Let's call
> it infinite integer (N_inf) over one infinity.

As far as I am concerned, "...999" does not mean anything at all. It has no connection with NSA or anything else as far as I can see. You keep throwing that string around as if you think it has a meaning, but you have never said what that meaning is.

I am not aware of any useful way of representing the members of *N as decimal digit strings. Any scheme I can think of suffers from one of two defects: either there are members of *N that don't fit into the naming scheme at all, or the coding scheme is so cryptic that you can't even compare the sizes of two numbers just by looking at the digits.

Perhaps you can think of some scheme that I have overlooked, but you will have to define your terms very carefully before I will be convinced.

> 2) I define it as I have done earlier sum (k 0 --> oo) 9*10^k.

That sum diverges, even in NSA. It is not a number.

> It does not
> have classic limit as the k refers now the standard point of reference, but
> it has a limit as you hopefully noticed as we have another point of
> reference.

There you go with your "point of reference" again. Sorry, but you can't use one undefined term to define another.

> 3) It is defined and it exists, now - how do You like to name it?
> Hyperinteger or something else?
> (snip)

It's a string of digits. Nothing more. It is not a number at all, at least in any sense that you have yet defined.

And now, I would like to ask a counter-question. I will describe a certain member of *N, and I would like you to tell me what decimal digit string you think corresponds to it. First, some background.

You may recall from other discussions that the (standard) real numbers can be defined as equivalence classes of Cauchy sequences of rationals. That means I can identify a real number by presenting you with a Cauchy sequence, and it is understood that any other sequence that happens to fall into the same equivalence class is an equally good representation of that number. For example, the sequences

< 9/10, 99/100, 999/1000, ... >

and

< 1, 1, 1, 1, ... >

happen to be two different representations of the same real number.

Now, let's consider a similar construction that lies at the heart of nonstandard analysis. To describe a member of *N, for example, I can present you with a sequence of natural numbers (members of N). Unlike the real−number construction, this one doesn't require the sequences to be Cauchy. Technically, I also need to describe to you the equivalence relation that will be used, but that's a bit more complicated. It involves something called a free ultrafilter on N. You can find an explanation of the concept at <http://mathworld.wolfram.com/Ultraproduct.html>.

For our purposes, it's enough to know that if I give you a sequence in N, there is a unique member of *N that is represented by that sequence. Ok so far?

Here is the sequence I have in mind. Let A(x,y) be the Ackermann Function, as described at <http://mathworld.wolfram.com/AckermannFunction.html>. Now, let a_k = A(k,k) for each k. This sequence starts out:

a_0 = A(0,0) = 1
a_1 = A(1,1) = 3
a_2 = A(2,2) = 7
a_3 = A(3,3) = 2^6 − 3 = 61
a_4 = A(4,4) = 2^2^2^2^2^2^2 − 3 = (too big to write out here)

and after that the sequence starts to grow rather quickly. :−)

Let a be the member of *N that is associated with this sequence. My question is:

(1) what decimal digit string do you think represents a?
(2) what decimal digit string do you think represents A(a,a)?

My point is that your decimal digit strings are woefully inadequate in this context. They cannot even begin to describe the numbers in *N in any useful way.

>>>> I'm trying to guess what you might mean by "the real part", since you
>>>> have not defined your meaning.

>>> the real part was the decimal part − as you certainly knew.

>> No, I didn't know what you meant, and I still am not sure. What is the
>> "real part" of sqrt(2), for example. It sounds like you trying to say
>> the "real part" is sqrt(2) − 1, or approximately 0.41421. That was not
>> one of my first two guesses. I would call that the "fractional part."

> Ok, that suits for me.

> (snip)

>> In a similar fashion, you keep assuming that I must know what you mean by
>> ...999. However, I assure you that I don't.

> Uh! I have tried to explain so simple as possible. I had once a fealing that
> You understood very well.
> There must be some miscommunication.

Answer my questions (1) and (2) above, and then we'll see whether there
is any purpose at all in discussing decimal digit strings in connection
with *N.

>> (4) It is not possible to sum over N in NSA, because N is
>> not a set in NSA.

>> Do you agree?

> I agree 1,2 and 3 but in the case 4 I disagree. Maybe we have to discuss
> about that topic more accurate. I consider finite integers are a subset of
> infinite integers over the first one infinity.

Each of the finite naturals is a member of *N, but it does not follow
from this that N is a subset of *N. That's part of why it's called
"nonstandard" analysis. Not everything is a set in this model.

>>>> The set N
>>>> (consisting of the finite naturals) is not a set in the internal set
>>>> theory of NSA.

>>> Coorect, but it should be as *N is the extension of the set N.

>> No, it should not be. There is an important principle involved, known as
>> the transfer principle. This says that every theorem of standard
>> analysis is also a theorem of NSA, provided we make the appropriate
>> substitutions (such as substituting *N for each occurence of N).

>> The transfer principle is extremely important. Without it, NSA loses
>> most of its value as a tool of analysis.

>> It turns out that if all sets in standard analysis are allowed to be
>> internal sets in NSA, then we lose the transfer principle. That's why
>> things are defined the way they are.

> I cannot see where the transfer principle fails, except in the case of sum
> and limit. But in this case there are natural reasons as I have tried to
> explain. And the reason is not NSA.

Let's consider the case of *R, for example. We know *R is the extension (the "enlargement") of R. We also know that R satisfies the least upper bound property. By the transfer principle, *R must satisfy the least upper bound property. However, we also know that the collection R of finite reals is nonempty and is bounded above in *R by any infinitely large number. But R does not have a least upper bound in *R, because if x is infinitely large, then so is x−1. The conclusion is that R cannot be a set in NSA. The least upper bound principle is preserved because it applies only to nonempty *subsets* of *R that are bounded above, and R is not a *subset* at all.

> (snip)

>>> Your opinion, because You could not consider the successor of ...999.
>>> What
>>> is the successor of ...999 as all the placeholders are infinitely
>>> occupied
>>> with the maximal digit 9?

>> You haven't said what ...999 is. How am I supposed to answer questions
>> about it if you don't define it?

> I assume You can now answer after the definition − above. I propose anyway
> to start a new thread.

You have not defined ...999. If you think you know what it is, then why don't you try answering your question and explaining your answer?

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Dave Seaman
Judge Yohn's mistakes revealed in Mumia Abu-Jamal ruling.
<http://www.commoncouragepress.com/index.cfm?action=book&bookid=228>