

Re: Cantor K.O.'d -- again! -- Further explication

Source: <http://sci.tech-archive.net/Archive/sci.math/2005-01/2049.html>

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Date: 01/07/05

Date: Thu, 06 Jan 2005 22:14:35 -0500

Mark Adkins wrote:

<snip>

>

> *By starting with a list consisting of one member, then adding additional members one by one, as the list grows in length, the length of the list members also grows, and the length of the diagonal grows with them; and it can be seen that the diagonal must grow *exactly* as the list members do, otherwise the diagonal cannot grow. The diagonal is a derivative entity, mirroring the growth of the list members.*

>

> *Is it possible for such a list to cover the range of natural numbers, N? This is equivalent to asking whether it is possible to have an L such that: (a) The quantity of list members is "infinite"; (b) The length of the diagonal is "infinite"; (c) The length of the members remains finite.*

>

> *Call this hypothetical list L*. If it exists, it may be regarded as a kind of limiting case for the process of list growth just described. In L* the diagonal is still made of the terminals of list members; however, the diagonal must now contain all possible terminal symbols.*

>

> *It may not always be explicitly recognized that a list is intrinsically a geometric entity: but this is the case, when each member is a string of symbols; each symbol occupies an ordinal location in an abstract space, and possesses spatial relations of "right" and "left" relative to its neighbors; and different members occupy similar ordinal locations, only their spatial relations, relative to one another, are those of "above" and "below"; and all of this implies that individual symbols and list members are separated by spaces so as to preserve their status as distinct entities. Thus, a "list" is a geometric structure. Metrics, however, being arbitrary, are rarely considered.*

>

- > *It is clear that L^* , if it exists, can be mapped into a unit*
- > *square in the plane. One such scheme, using standard*
- > *orthogonal coordinates, maps each symbol according to*
- > *the following formulae:*
- >
- > $x = 1 - 1/2^{(m-1)}$
- > $y = -1 + 1/2^{(n-1)}$
- >
- > *It is understood that the mapping specifies these points*
- > *as the centers of the circular symbols.*
- >
- > *Thus, the symbols of the diagonal, running from upper-left*
- > *to lower-right, take the coordinates:*
- > $(0,0)$; $(1/2, -1/2)$; $(3/4, -3/4)$; $(7/8, -7/8)$; etc.
- >

---Emphasis Added:Begin

- > *The coordinates of the unit square's corners are:*
- > *UL $(0,0)$; UR $(0,1)$; LL $(0,-1)$; LR $(1,-1)$. It can be seen that*
- > *except for the Origin at the upper-left, these are limit points*
- > *and do not have list symbols mapped to them.*

--- Emphasis Added:End

<more snip>

- > *we will be*
- > *taking a mind's-eye tour, travelling along the diagonal line*
- > *connecting $(0,0)$ with $(1,-1)$, stopping at each of the diagonal's*
- > *mapping points at each stage of its construction, to make*
- > *sure the job is done right. Our average travel speed is a*
- > *leisurely square-root-of-two units per hour: thus we shall*
- > *traverse the entire diagonal in one hour. Those who maintain*
- > *that it is impossible to traverse an infinite number of points in*
- > *a finite time should consult Zeno; and those who remain skeptical*
- > *should take this opportunity to announce their rejection of*
- > *point-set geometry and the calculus.*
- >
- > *Our little day-hike of infinity may thus be considered illustrative*
- > *of the adage solvitur ambulando. (Despite Randy Poe's hysterical*
- > *invective to the contrary, this does not properly translate as "proof*
- > *by foot-stomping", even if it does entail a kind of forced march.)*
- >
- > *Our tour originates at the Origin. Here, the first symbol of the*
- > *diagonal is laid. It is also, of course, the first (and only) symbol*
- > *of the first list member. At this point, the diagonal and the first*
- > *list member are identical.*
- >
- > *On we go: The second symbol of the diagonal is laid. The*
- > *diagonal, now consisting of two symbols, is identical to the two*

- > symbols of the second list member. That is, in both length
- > and content, the diagonal and the second list member are
- > identical.
- >
- > Here we are at the third stage. The diagonal and the third list
- > member are now identical. The pattern is clear. It is, I fear,
- > rather monotonous. We shall skip ahead some considerable
- > way...
- >
- > Here we are at the 1,013_{th} stage. The diagonal is identical,
- > in both content and length, to the 1,013_{th} list member.
- > Goodness, this is boring! When *will* the diagonal become
- > infinite, thus distinguishing itself from the list members, all
- > of which must remain finite? Hmm...that's a toughie. The
- > Cantorists would probably answer: "When the list is completed".
- > We shall see.
- >
- > Meanwhile, *how* can the diagonal ever become infinite, since at
- > each stage n the diagonal is identical to the n _{th} list member,
- > and every possible n is a finite natural number?
- >
- > The diagonal is a derivative entity, and merely mirrors the list
- > members from which it is derived. It *should* therefore be
- > clear that in order for the diagonal to become infinite and
- > contain all possible symbols, some eventual list member must
- > itself become infinite and contain all possible symbols. After
- > all, the diagonal we are constructing can grow no faster -- and
- > no longer -- than the list members from which it is derived. Well,
- > let's address the matter again when we are done...
- >
- > ...And here we are, after one hour of travel, at coordinates $(1, -1)$,
- > the bottom-right corner of the unit square. Have we not constructed
- > every member of L^* using our straightforward mapping formulae?

No. As you yourself stated, in the "Emphasis Added" section, one doesn't in fact reach the coordinates $(1, -1)$, since no point (i.e. no natural number) is mapped to it. One can get arbitrarily close, but your one hour of travel won't get you there; there will always be another member of L^* which you haven't mapped yet.

The idea which you state in that section, that the point $(1, -1)$ is a "limit point" of your mapping, implicitly assumes the idea of an infinite collection of points in a finite amount of space. It looks to me as if you've just constructed the list whose impossibility you've been asserting. Congratulations.

Matt