

# basic linear algebra determinant proof

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This is in the context of numerical analysis, and the Gaussian Elimination with backward substitution algorithm.

The proof seems very easy, and so I am wondering if maybe I skipped some steps that I shouldn't have, or maybe this is just a freebie :)

If a nonzero pivot element cannot be found at any stage, then the matrix is singular.

Proof:

Let  $A$  be a matrix where a nonzero pivot cannot be found at some stage. Write  $A$  in upper triangular form by applying row operations. Since a nonzero pivot could not be found at some stage, there is a zero element on the main diagonal of the matrix. Then, because the determinant of a matrix in upper triangular form is the product of the elements on its diagonal, the determinant is zero. And thus by the "Determinant Criterion for Invertibility" Theorem, the matrix is singular.

Aside, could I also argue as follows? Write  $A$  in row echelon form. Since a nonzero pivot could not be found at some stage, there exists a row with all zero elements, and thus  $A$  is singular.