

Re: Limit of function

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ticbol wrote:

> *Robert,*

>

> $\lim_{x \rightarrow \infty} \ln(x^2 - 2x + 1) / \ln(x^{10} + 3x + 1)$

> $x \rightarrow \infty$

>

> = $(\infty)/(\infty)$

> *Indeterminate again.*

>

> *Use L'Hopital's Rule again.*

>

> $f(x) = \ln(x^2 - 2x + 1)$

> *So,* $f'(x) = [1/(x^2 - 2x + 1)] * (2x - 2)$

> = $(2x - 2)/(x^2 - 2x + 1)$

>

> $g(x) = \ln(x^{10} + 3x + 1)$

> *So,* $g'(x) = [1/(x^{10} + 3x + 1)] * (10x^9 + 3)$

> = $(10x^9 + 3)/(x^{10} + 3x + 1)$

>

> *Then,*

> $\lim_{x \rightarrow \infty} \ln(x^2 - 2x + 1) / \ln(x^{10} + 3x + 1)$

> $x \rightarrow \infty$

>

> = $\lim_{x \rightarrow \infty} [(2x - 2)/(x^2 - 2x + 1)] / [(10x^9 + 3)/(x^{10} + 3x + 1)]$

> $x \rightarrow \infty$

>

> = $\lim_{x \rightarrow \infty} [(2x - 2)(x^{10} + 3x + 1)] / [(x^2 - 2x + 1)(10x^9 + 3)]$

> $x \rightarrow \infty$

>

> = $\lim_{x \rightarrow \infty} [2x^{11} + 6x^2 + 2x - 2x^{10} - 6x - 2] / [10x^{11} - 20x^{10} + 10x^9 + 3x^2$

> $- 6x + 3]$

> $x \rightarrow \infty$

>

> = $\lim_{x \rightarrow \infty} [2x^{11} - 2x^{10} + 6x^2 - 4x - 2] / [10x^{11} - 20x^{10} + 10x^9 + 3x^2 - 6x$

> $+ 3]$

> $x \rightarrow \infty$

>

> Divide both numerator and denominator by x^{11} ,
>
> = $\lim [2 - 2/x + 6/x^9 - 4/x^{10} - 2/x^{11}] / [10 - 20/x + 10/x^2 + 3/x^9$
> $- 6/x^{10} + 3/x^{11}]$
> $x \rightarrow \text{inf}$
>
> = $[2] / [10]$
>
> = $2/10 = 1/5 = 0.2$ -----answer.

Typical "mathematics made difficult" :-(

$$\log(x^2 - 2x + 1) = \log(x^2) + \log(1 - 2/x + 1/x^2) = 2 \log x + O(1/x)$$

similarly

$$\log(x^{10} + 3x + 1) = 10 \log x + O(1/x^9).$$

Hence

$$\log(x^2 - 2x + 1) / \log(x^{10} + 3x + 1) = 2/10 + O(1/(x \log x)) \rightarrow 1/5$$

as $x \rightarrow \text{infinity}$.

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Robin Chapman, www.maths.ex.ac.uk/~rjc/rjc.html

"Lacan, Jacques, 79, 91-92; mistakes his penis for a square root, 88-9"

Francis Wheen, [_How Mumbo-Jumbo Conquered the World_](#)