

Well-ordered series of ordinals

Source: <http://sci.tech-archive.net/Archive/sci.math/2005-01/3177.html>

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Date: 01/11/05

Date: 11 Jan 2005 04:04:09 -0800

Given a family (a_i) of ordinals indexed by a well-ordered set I .
Define the sum $\sum_i a_i$ as the unique ordinal isomorphic to the well-ordered set:

$$X = \bigvee_{\{i \in I\}} \{i\} \times a_i$$

where the good order on X is given by:

$$[(i,x) \leq (j,y)] \iff [(i < j) \vee ((i=j) \wedge (x \leq y))]$$

I would like someone to confirm that provided I is isomorphic to a limit ordinal, we have the equality:

$$\sum_i a_i = \sup_{\{J < I\}} \sum_{\{i \in J\}} a_i$$

where $J < I$ indicates that J is a strict initial segment of I .

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