Re: THIS STATEMENT HAS NO PROOF IN ANY SYSTEM = true or false?

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From: Ralph Hartley (*hartley_at_aic.nrl.navy.mil*)
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tchow@lsa.umich.edu wrote:
> Ralph Hartley <hartley@aic.nrl.navy.mil> wrote:
>
> I wasn't sure to what extent you were defending the
> earlier posters in this thread

To be honest, neither was I.

I have spent way more time on this thread than I should have, and learned more than I expected. But I can't continue much more.

> But in any case, the real issue again is your assumption that
> mathematicians use an effective procedure to determine mathematical truth.
> As I've argued elsewhere in this thread, CT theses of whatever flavor
> don't yield this assumption; CT theses will only say that if mathematicians
*> are indeed using an effective procedure to find, or are *computing*,*
> mathematical truths, then the theory of [partial] recursive functions
> applies and lets us deduce further conclusions. But it doesn't say
> whether mathematicians are indeed using an effective procedure.

This argument is not without merit. I don't *think* I totally agree, but I don't have time to decide for sure, or to explain my problems with it much more than I already have.

It may not be enough, for your argument, for the procedure to not be effective, it needs to be *better* than any effective procedure. In particular it needs to be both sound and complete.

Otherwise you will come to "know" false things, or there are true things you will never know.

If a mathematician in a particular universe can correctly answer all members of a class of questions (by any means, effective or not), and there is any recursive process to determine what she has concluded, and my version of the physical CT thesis holds for that universe, then the class of questions is recursive.

> I know of nobody−−−atheist or believer−−−who thinks that
> theological doctrines are generated by an effective procedure.

I'm not sure I would go so far as to say that I *think* that, but I see no reason to conclude that it isn't. There are effective procedures for producing nonsense.

Some believers consider reference to a particular text to be the first, last, and only way to obtain truth. Looking up the answer in a book seems pretty effective to me.

> You've argued that if something is obtained by a procedure that is not
*> effective, then it's not *knowledge*. Perhaps that's true, but the*
*> the corpus of what is *generally called* "mathematical knowledge" is*
> not generated by an effective procedure in any obvious way. "Every
> vector space has a basis" is generally considered to be a mathematical
> truth. It can be proved using various axioms, including the axiom of
> choice. But how did we come to accept the axiom of choice?

I'm not sure I would call "Every vector space has a basis" a mathematical truth if that is what you really meant when you said it. I think I *would* call "In ZFC every vector space has a basis" a mathematical truth.

But you are presumably a mathematician, and when mathematicians make unqualified statements, with no other context, they usually *mean* "In ZFC ...".

In a sense, mathematical axioms are not knowledge because they are not statements that can be true or false in an absolute sense. They can be viewed as being more like definitions.

The axioms of group theory are not facts that are "known", they describe what we *mean* when we talk about a group.

There was once quite a bit of fuss over the truth of the Parallel Postulate. Nowadays, we would call it a property of a space. It isn't true or false in an absolute sense. Some spaces have it and some don't.

Similarly, accepting AC can be viewed as being more specific about what you mean by "sets".

I am not sure I am willing to follow this line of reasoning to its ultimate destination. I like to think I know what I mean by *the* integers, and there are some statements that I might have trouble viewing as "a matter of definition", even though they are independent of all the axioms.

I imagine people used to feel that way about points and lines.

I really am quite fond of that "little white lie".

> *There were heated arguments, and through a complex sociological process, the axiom*
> *of choice won out. Was this an effective procedure? Was it even objective?*

Which is exactly what one would expect if it was a matter of definition, not of truth. Truth isn't normally considered a matter of consensus, and is not considered negotiable, but definitions are.

Definitions can be produced by an effective procedure or not, because there is no need for them to be objective.

There are good definitions and bad ones, but it is mostly a matter of utility. Asking if a definition is *true* is nonsense (we can, and should, ask if a definition is consistent).

> *What's to stop some similar mess from happening again−−−say, with the axiom*
> *of projective determinacy, which Woodin and others have been advocating?*

It most certainly will, if not in that case, then in some other.

> *If these things aren't knowledge, or aren't mathematical truths, then*
> *your assumption comes at the cost of discarding a lot of what most people*
> *consider to be mathematical truth.*

In the case of set theory, that would be less than one page of text. Including definitions might require a small font.

The axioms themselves are a very small part of mathematics. Most (one could argue all) mathematical knowledge is in the theorems.

One way to evaluate a mathematical theory is to look at its "theorem to axiom ratio". All else being equal, bigger is better.

A big bunch of complex axioms, from which little additional can be proven is barely mathematics. We like small a set of simple axioms with an enormous number and diversity of theorems (e.g. set theory).

"Good" theories are more useful. The small set of axioms mean they apply more often, and the large set of theorems mean you get a lot of answers.

I suspect that one reason that AC is accepted is that it is simple and produces many diverse theorems.

It is unreasonable to expect to get an answer to *every* question. Some of the *best* theories allow one to express questions that they cannot answer.

You can always *define* answers to questions that your axioms don't answer, but it is unclear in what sense that is the same as knowing the answer, and there in no sure (or even effective) way to avoid making inconsistent assumptions.

If you keep adding axioms, without proving then consistent, you will surely
add an inconsistent one eventually.

Some statements cannot be proven "safe" to use as axioms (even if they
are). (The safe axioms are the complement of an r.e. set) If you only add
safe axioms, there are some statements you will never decide.

AC is a not the best example, since it *was* proven safe.

Ralph Hartley