Re: Rings and F−Algebras

Source: http://sci.tech−archive.net/Archive/sci.math/2005−01/6072.html

---

From: Stuart M Newberger (*smnewberger_at_comcast.net*)
Date: 01/20/05

```
Date: 20 Jan 2005 13:33:47 −0800
```

Porker899 wrote:
> *Right? Wrong? Way off? I would appreciate help or a proof.*
>
> *Definition of F−Algebra: A ring R that is a vector space over F with*
the same
> *addition and ring multiplication and scalar multiplication are*
related by
> *(theta*a)*b=theta(a*b)=a(theta*b) for a,b in R and theta in F.*
>
> *Problem: Let R be a finite dimensional F−algebra where F is a field.*
If Ris
> *nontrivial and if r,s belong to R with rs=0 then either r=0 or s=0*
IMPLIESS R
> *is a division ring.*
>
"Proof": For r not zero in R define the function s −−> s*r is an
F−linear map from R to R. If
s*r=0 implies s=0 in R then there are no nonnzero elements in its
kernal.
Therefore the map is also surjective (onto R,since R is a finite
dimensional vector space over F). Thus it is a division ring.

Hold the phone.If you think that the axioms of a ring include a
multiplicative identity 1 then you have shown that given r not zero
that there is u with ur=1 and similarly (using the map s−>rs) that
there is v with rv=1 .A well known one liner shows u=v and we are done.

What if you do not assume R has an identity .Then you must prove that
there is an identity in your ring with no zero divisors.Here is a hint
for this.f is any r in R not zero then by the above you can choose s
with rs=r s is not zero .Show s is an identity. 2nd hint: You can
choose t with st=1.Show using that R has no zero divisors that s=t
.Thus ss=s Now Every b in R is of the form b=sc (c−>sc is onto) so that
sb=ssc=sc=b (similarly bs=b) and s is the identity for R.
Regards,Stuart M Newberger