

Re: Need hints for a problem about primes.

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From: Don Taylor (*dont_at_agora.rdrop.com*)

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"Snis Pilbor" <snispilbor@yahoo.com> writes:

>I am clawing my hair out trying to figure this out. The problem
>is to prove that if a^{n+1} is prime ($a > 1$) then n is a power of 2. I
>honestly don't even know where to start. Certainly this would imply
> $a^n \equiv -1 \pmod{p}$ [where prime p is just a^{n+1} , but we "forget" this
>relationship with a and just focus on the fact that a^n is $-1 \pmod{\text{'some$
>prime'}], and this is the only starting point I can think of, but this
>is a long long way away from a solution and I'm hopelessly stumped of
>even what broad, general strategy to use. Other grasps in the dark
>include observing that $a^{2n} \equiv 1 \pmod{p}$, and if we can show $2n$ is a power
>of 2 we're done, but this too leads nowhere; and that we can safely
>assume a is itself not a nontrivial power of anything (or just pull
>that power into n). Obviously a is even.

>Fair warning: this problem DOES arise amid homework. I'm just
>hoping for a hint or clue...

Well, what if n were odd? Can you get anything from that?
Can that let you make a tiny bit of progress.