Re: On Well−Ordering(s) and Sets Dense in the Reals, Infinity

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Ross A. Finlayson wrote:
> *About the infinite binary sequences, there are definitely*
> *considerations to be made about the density of the elements, and*
> *restrictions on the sequence element interchange in permutation.*

Sure, I was just pointing out one way in which saying "all infinite sets are equivalent" can end up with very counterintuitive results.

> *I'm hoping you would briefly summarize, in an obvious way, the*
> *functions you mention not defined over the reals*

A single example of many: in one application I was dealing with measures of sets of functions from certain grammars to natural numbers. Some of the grammars gave rise to uncountable sets of functions, some gave rise to countable.

Of course, in a system where "infinite sets are equivalent" it would have been pointless, because the infinite sets I was dealing with would all have been equivalent.

> *I have just been very casually examining the measure theoretical*
> *foundations and have not seen non−geometric examples.*

Measure theory may have started with geometric motivation (I'm not sure of its history), but if so then it has certainly grown far beyond that.

> *You say all of analysis is under measure theory, or that it's*
> *expressible in terms of measure theory.*

Not exactly; I just said that if you're going to retrofit measure theory to fit an "all infinite sets are equivalent" axiom, you'll need to retrofit all of analysis just to begin with.

> *A lot of people do integral calculus without regard of measure*
> *theory.*

A lot of people balance their books without regard of the abelian group theory underlying addition of integers, either. However, if addition were no longer commutative, associative, or had inverses then the calculations wouldn't balance because they use them *implicity*.

− Tim