Re: THIS STATEMENT HAS NO PROOF IN ANY SYSTEM = true or false?

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From: Jeffrey Ketland (*ketland_at_ketland.fsnet.co.uk*)
Date: 01/27/05

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Date: Thu, 27 Jan 2005 14:47:57 −0000
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poopdeville@gmail.com wrote in message
>tchow@lsa.umich.edu wrote:
*>> First of all, the way mathematicians *in fact* use the word "true"*
>> violates your norms. They may be guilty of a philosophical
>transgression,
>> but if you want to understand what they're saying, you have to get
>used
>> to this usage whether you like it or not. I think this is how the
>whole
>> (sub)thread went flailing off in the first place: You just weren't
>used
>> to normal mathematical talk.
>>
>
>Of course. I addressed this in the paragraph you snipped. However,
>normal mathematical talk is still quite confused. Here I refer to your
>(and Jeffrey Ketland's, but mostly his) talk of disquotational schemes,
>with emphasis of Jeffrey's use of models and the real world and your
>group theory example at the end of your message.

Sorry. I've snipped the rest, but this really is the heart of the matter. Mathematical talk is *not confused*. Rather, you seem to be advocating some unmotivated, and possibly incoherent, form of scepticism.

Consider Goldbach's Conjecture, GC. Its truth condition is stated as follows:
GC is *true* if and only if for every even number n, there are primes p1 and p2 such that n = p1 + p2.

At present, we do not know if GC is true or false. But this is a precise analysis of what saying "GC is true" means.

The above is an instance of the partial defintion giving what the word "true" means. It illustrates how the word "true" is actually used, both in ordinary life, in science and in mathematics. Truth for interpreted statements is intrinsically disquotational, just as 7 is intrinsically

prime. This has nothing to do with the "redundancy theory", which Tarski refuted. It is a central property of the notion of truth.
(Proof: Let (L, I) be an interpreted language and let T be the set of truths in (L, I). Suppose that dom(I) contains all the expressions of L. Suppose that L also contains a predicate True(x) which defines this set T. For each expression E, let E* be a term in L such that E* denotes (in I) E. Then, for any A in L, each sentence True(A*) <−> A is true in (L, I). Disquotation is an intrinsic property of truth.
More generally, if (ML, MI) is a meta−language for (L, I), and True(x) is a formula in ML which defines truth in (L, I)) and there is a translation t which maps L−expressions to ML−expressions, then we get that True(A*) <−> t(A) is true in (ML, MI), for each sentence A of L. This is Tarski's Convention T.)

We have a precise and exact mathematical theory of truth (for arithmetic). The set of arithmetic truths is precisely defined. Suppose we take L_{PA} with just ~, & and "forall" as primitive logical expressions.

Tr(N) is the smallest set X such that
(a) X is a subset of Sent(L_{PA})
(b) for any closed terms t, u, t=u is in X iff val(t) = val(u)
(c) for any A in Sent(L_{PA}), ~A is in X iff A is not in X
(d) for any A, B in Sent(L_{PA}), A&B is in X iff A is in X and B is in X
(e) for any A, for any v, "forall v,A" is in X iff, for any n in N,
A(n/v) is in X

This set is Sigma^1_1. It is not definable in arithmetic. Etc.
This body of mathematical work began in the 1930's with Alfred Tarski and has developed a great deal since (Mostowski, Feferman, Kripke, Friedman, etc.). If you have a precise objection to this serious and correct analysis of truth, then what is your objection?

Of course, one can also talk of truth for *uninterpreted formulas*, using the notion of truth−in−an−interpretation (NOT model−−−this assumes you have some axioms around), also defined by Alfred Tarski. This concerns uninterpreted formulas and structures, and whether such a formula is true relative to that structure. Thus,
Fab is true in I if and only if (a^I, b^I) is in F^I.
Note that semantic theory itself is riddled with set theory. F^I is, for example, a subset of the Cartesian product D^2 where D is the domain (i.e., a set) of I.

But GC is not an uninterpreted formula. It is a meaningful statement about the numbers.
Your argument requires that we literally identify a meaningful statement and its logical form (the associated uninterpreted formula), thereby denying that mathematical statements are meaningful statements.
I can think of no good reason for doing this. Not even nominalists do this (they simply deny the existence of numbers, sets, etc., tout court).
To illustrate, let GC be Goldbach and let GC* be its logical form. Then we have:

GC: For any even number n>2, there are primes p and q such that n = p + q

GC*: Ax(Fx & Rxa −> EyEz(Gy & Gz & x = f(y,z))

Similarly, from baby logic, we have things like:

S : Lennon is taller than McCartney
S*: Pab

By the way, that it isn't hard to prove that
GC is true if and only if N |= GC*
Similarly, in the semantical meta−theory for the language of ZF, we can prove
AC is true if and only if V |= AC*
as well as
AC is true if and only if every set of the right sort has a choice set.

If you want to argue that mathematical statements are meaningless (and thus should be identified with their uninterpreted logical forms), then you really need to give a precise argument for this radically sceptical claim.
I see no relevant difference between GC and S. Whether GC is true depends upon the properties of even numbers and primes; whether S is true depends upon the properties of John and Paul.

Furthermore, if you think that ordinary mathematical statements are meaningless, but also that meta−mathematical statements about models are meaningful, then you seem to be contradicting yourself, as Tim pointed out. Indeed, what is a model but a set?

(A set−sized structure for the language of ZF is a pair (D, R), where D is a non−empty set, and R is a subset of the set D^2. A structure (D, R) is a model of ZF if and only if all axioms of ZF are true in (D, R).
This is why one cannot intelligibly define "set" in terms of "model of ZF". It is incoherently circular. In contrast, one can define "group" in terms of "model of group axioms G1, G2, G3". Thus, (D, o) is a group iff D is non−empty set and o is a binary associative operation on D, with a unit, unique inverse, etc.)

Also, how exactly would you define "A is true in M" without a theory of sequences, etc.?
Come to think of it, if you're advocating some sort of radical scepticism about meaning, what is the "intended interpretation" for this post to sci.logic?
E.g., why are statements about numbers meaningless, but, say, "Jeff was born in England" meaningful?
Everyone agrees that semantical theory is difficult (e.g., in natural languages there is ambiguity, vagueness, indexicality, intensionality, etc.), but what you are saying about the semantics of mathematical statements doesn't make much sense. For another example, on your account, we cannot even deal with minimal *applications* of mathematics to the physical world, as in "The number of elephants in London Zoo is exactly 5" or "The

axial−vector function that represents the magnetic field has zero
divergence".

−−− Jeff