

# Tough Integrals & A Strange Distribution

**Source:** <http://sci.tech-archive.net/Archive/sci.math/2005-01/8566.html>

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**Date:** 01/29/05

Date: Sat, 29 Jan 2005 21:07:52 +0100

Hi all,

here a simple but difficult question:

Does the following probability density in  $x$  and  $y$

$$f(x,y) = C(a,b) * ((1 + a*(x^2+y^2)) * (1 + b*(x^2+y^2))^{-1})$$

represents a special form of a bivariate Student-t distribution?

Above,  $a,b>0$  and  $C(a,b)=\ln(a/b)$  is the normalization constant.

For the special case  $a=b$ , it is easily seen that  $f(x,y)$  is bivariate  $t$  (with 2 degrees of freedom);

...if  $a$  is different from  $b$ , is it still a  $t$ -distribution ??

I'm asking because I want to compute the two-fold probability integral  $P(x<X,y<Y)$ . The first integration is not too difficult, but the second seems impossible. It would be good to know whether mathematicians have already defined the spherical density from above.

Here some forms the integrals to be solved can have:

- 1)  $\int \ln(\sin^2(x) + a) dx$
- 2)  $\int \arctan(x) / (x*\sqrt{1-x}) dx$
- 3)  $\int 1/\sqrt{1+x^2} * \arctan(1/\sqrt{1+x^2}) dx$

Using Maple on 1) the gives a loooooong and unpleasant result in terms of the dilogarithm, not very helpful at all...

Any help is welcome!  
Christian.