Re: The generality of mathematics

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In article <ctjn76hpj1@gemini.csx.cam.ac.uk>,
Jamie Vicary <jamievicary@gmail.com> wrote:

> Is mathematics completely general? Can all possible algebraic
>structures be represented by the mathematical structures which we use
>today?

Depends entirely on what you mean by "algebraic structures"! Given certain definitions, then Universal Algebra covers all possibilities. But given other definitions, it does not.

> It seems to me that modern mathematics is not really very general
>at all. Much of algebra is dominated by the notion that objects can be
>operated on from the left, and from the right. Why not other
>"directions"? Why not conceive of a set of objects which operate on each
>other in a much more general sense?

You may find the recent work of Vaughan Jones on what he calls "planar algebras" interesting and relevant to what you are talking about (as I understand it, though I haven't looked through his stuff very carefully, he considers the possibility of operators from "above" and "below" and other directions in addition to the usual left and right.

http://math.berkeley.edu/~vfr/
http://math.berkeley.edu/~vfr/plnalg1.ps
http://math.berkeley.edu/~vfr/delphi.ps

> Most importantly, does there exist a proof that ANY possible
>structure between a set of objects is equivalent to some structure that
>can be formed using the formalism of modern mathematics?

In order for such a proof to exist, you would have to provide a formal definition of "possible structure between a set of objects". But doing that begs the question, because what you are really trying to capture is an intuitive notion. In short, you would be facing the exact same problem that logicians and computer scientists have faced with regards to the notion of "algorithm": we have an intuitive notion of what an algorithm is; we have a (number

of) formal definition(s) that try to capture that idea. It is not hard
to verify that everything that is encompassed within the definition is
indeed an algorithm, but it is impossible to prove that everything we
would consider an algorithm would fall under that definition (though
it is possible to prove it does −>not<−, by exhibiting such an
algorithm and showing it does not fall within the bounds of the given
definition). Look up "Church's Thesis".

So you are asking about an analogue of Church's Thesis for the notion
of "structure between a set of object".

There are already definitions in place for the notion of "algebra",
both finitary and infinitary, partial and complete; they include
structures ranging from grupoids and semigroups to topological spaces;
there is Vaughn Jones work on planar algebras that extends those
notions. Whether or not this encompasses everything that you would
recongize as a "structure between a set of objects" is a statement
trying to relate an informal, intuitive idea with a formal one. There
is no way to prove it correct, though you could prove it wrong by
exhibiting a structure that does not fall within those bounds.

> If such a proof
>does not exist, why have we not developed a branch of mathematics which
>CAN, in principle, deal with all conceivable types of structure between
>objects?

Look up "Universal Algebra".

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"It's not denial. I'm just very selective about
 what I accept as reality."
    --- Calvin ("Calvin and Hobbes")
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