Partitions of unity question

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If M is a topological space with the property that for every open cover X of M, there exists a partition of unity subordinate to X, show that M is paracompact.

I can't quite see how to do this. If {f_i} is a partition of unity for X = {X_i}, then certainly

X = Union of {supp(f_i)} (over i), since if x is in M, then there exists a function f_k such that f_k(x) is not zero since by definition of parition of unity,

sum f_i(x) = 1 for all x in M (where the sum is taken over all i).

So if supp(f_i) was open for all i, then {supp(f_i)} would be a locally finite refinement of {X_i} since supp(f_i) is a subset of X_i and the supp's form a locally finite set.

But they may not be open. So I'm not sure what to do. For a particular k, can I maybe write supp(f_k) as a union of a finite amount of open sets? Because it seems clear to me that a finite refinement of a locally finite set is locally finite. But it doesn't appear clear to me that any refinement of a locally finite set is locally finite.

Any help is highly appreciated, Thank you,

Tony