Re: Epistemology 201: The Science of Science

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> >> You are perhaps referring to First Order Predicate Calculus (FOPC).
> >> And indeed, mathematicians do use FOPC. However, mathematics is not
> >> FOPC, and FOPC is not sufficiently expressible to allow it to be used
> >> exclusively.
>
> >> Given a particular system of axioms, say PA (the Peano Axioms),
> >> mathematicians could in principle use FOPC applied to those axioms.
> >> But mathematics is not confined to working within a particular axiom
> >> system. Moreover, the discussion axiom system itself is part of
> >> mathematics.
>
> >Maths is an extension of FOPC, like PA.
>
> Not really. Mathematics is much older than FOPC, so it doesn't make
> sense to say it is an extension of FOPC.

Okay, this is really strange to me because this is so not what I've come to understand mathematics as. These days, in mathematical reasoning, logical arguments are used to deduce consequences (theorems) of the assumptions of maths (axioms). Most of maths is built from sets, so the basic assumptions of maths are the axioms of set theory, in particular ZFC set theory. [Chapter Zero − Fundamental Notions of Abstract Mathematics, Carol Schumacher]

You are suggesting that maths is not this formal system, so I am lead to assume that you have some sort of prior understanding of what is mathematically legal and illegal, like most people. But is this type of reasoning informal or have we our own set of assumptions, much like axioms, that enable us to perform mathematical inference. When there is a disagreement, where do we turn? From my understanding it is this formalised system of mathematics, which took root with Whitehead and Russell in the principia mathematica. Hence its FOPC roots.

I can accept that the axioms are not often invoked in the heat of proofs, but then neither is the road−code when we are driving. Axioms as such don't need to be the way to go either. The more intuitive way to go are to use rules of inference, which are equivalent and perhaps closer to the story about how we 'do' maths.

Out of interest, if maths is not this formal system then how can abstract
mathematics take place? For example, how can the continuum hypothesis be
(dis)proven, or proved not to be provable?

> >and assumed, as far as I am aware.
>
> Again, not really. Mathematicians often try to make do with minimal
> axioms.

Which ones? The choice is critical to what is provable and what isn't.

> > If another system is used in maths then
> >people need to know about it. The ZF system without the axiom of Choice for
> >example, can lead to the creation of two spheres out of one in topology.
>
> I'm not sure of your point there.
>
> If you happen to be making a vague reference to the Banach−Tarski
> paradox, then you have it wrong. Banach−Tarski does depend on the
> axiom of choice.

I went to a seminar on this last year and I thought the dude said the problem
went away with invoking the axiom of choice. But now having read some more I
realise I misunderstood. Okay, bad example.

How about another then. It has been proven that in ZFC set theory, the formal
system of mathematics (I honestly can't see why you flatly refuse that there is
such a system), the continuum hypotheses is can neither be proven or disproven.
So it could be asserted true or false with a new axiom and there would be two
overlapping but distinct mathematical universes to choose from.

If ZFC is assumed as the foundations of maths, it has been shown by Chaitin that
there are infinite arithmetic truths that cannot be proven in ZFC. Where does
maths as not−a−formal−system fit into this?

> >The study of axioms don't take place in maths. It is meta−logic or
meta−maths
> >that deals with this. Godels theorem for example is a meta−mathematical
proof.
>
> While Goedel's theorem is meta−mathematics, nevertheless a lot of
> mathematics is effectively a study of axioms and their consequences.

This cannot be if maths is a formal system. But I understand that you don't tak
e it to be one and this statement is contingent on this.

> >> >Since mathematics has evolved along−side science and plays a large part in
> >> >describing and predicting how the world works, then as a formal system
goes,
> >it
> >> >seems to be on the money as far as capturing something about the world.

>
> >> That's your opinion. As a mathematician, I have a different
> >> opinion. I consider it important that mathematics is not about the
> >> world. Roughly speaking, mathematics is about what would happen if
> >> reality did not intrude. We discover a lot about reality by seeing
> >> how it differs from the mathematical ideal.
>
> >Fair enough. The formal system of maths is ripe for exploration. People
study
> >it divorsed from the world. But why spend so much time on maths and not some
> >other formal system? I think because of the close link maths has with the
> >world.
>
> There you go again. You talk about "the formal system of maths", but there
> is no such formal system. Then you suggest that we should instead
> study some other formal system. It is gibberish.

Is ZFC set theory a small and inconsequential part of mathematics? I suppose you don't really get into it unless you study number theory, mathematical logic and stuff, but it was my understanding that this system what the foundation of modern maths.