Re: Is 0.123... a member of the set {0, 0.1, 0.12, 0.123, ...}?

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I could easily add some repeating decimals and irrational decimals to these sets to temporarily show the spuriousness of your argument as written − but only until you used more precise language concerning the pertinent sequence within the set. Nevertheless, I recognize your objection as legitimate and appreciate the mental exercises it created for me. I am still having difficulty understanding how the elements in the Set Bn which are all contained in Set An somehow stop being contained in Set An as n => oo. Is my representation of Set Bn as n=> oo incorrect?

Here are some ramblings that I hope allow you to see my difficulty in understanding this matter.

Consider the following series of paired sets Ai and Bi.

For n = 1,
Set A1 = { 0, 0.1 }
Set B1 = { 0.1 }
Set B1 is a subset of Set A1.

For n = 2,
Set A2 = { 0, 0.1, 0.12 }
Set B2 = { 0.12 }
Set B2 is a subset of Set A2.

For n = 3,
Set A3 = { 0, 0.1, 0.12, 0.123 }
Set B3 = { 0.123 }
Set B3 is a subset of Set A3.

For n > 3
Set An = { 0, 0.1, 0.12, 0.123, ..., 0.123...n }
Set Bn = { 0.123...n }
Set Bn is a subset of Set An.

In both sets An and Bn, the "n" represents the decimalic placeholders

and digits occupied by the integer n and the "... " within the number "0.123...n" represents consecutive integers (if any) greater than 3 and less than n.

An infinite set A* is formed corresponding to n => oo
Set A* = { 0, 0.1, 0.12, 0.123, ..., 0.123...n, ... }
Set B* = { 0.123... n... }

Note: Set A* has no largest n and no last n.
Set B* has no largest n and no last n.

As n => oo, for all n, every element Bn is found in An.

Set B* properly represents the set Bn for n => oo. Is that correct?

The infinite Set A* may be more simply represented as { 0, 0.1, 0.12, 0.123, ... } in the same manner as the infinite set of naturals may be more simply represented by { 0, 1, 2, 3, ... } without recourse to an "n".

Wth respect to Set A*, "0.123, ..." is an equivalent representation of "0.123...n, ..." where the "...n, ..." is replaced by the ", ..." to indicate that the rational series associated with n => oo for the Set A* (whose limit is the irrational number 0.123...) is an infinite set of rational numbers with no largest and no last element. Here the "0.123, ..." indicates that the series of rational elements (which are in fact a series of numbers whose limit is the irrational number 0.123..) continues without end and cannot be completely specified, but the series is clearly infinite.

Wth respect to Set B*, 0.123... is an equivalent representation of 0.123...n... where the "...n..." is replaced by the "..." to indicate that the number represented by the element in Set B* is an infinite decimalic representation (with no last digit holders associated with the neverending placement of the next integer). While the representation 0.123... of the element in Set B* may appear to be the well defined irrational number, in fact from its definition, it is merely a representation of the ongoing process (as n => oo) of placing the next consecutive integer in the placeholders to the right of the previous decimalic number. As such each and every rational decimalic number obtained by the infinite process represented by the 0.123... element in Set B* is a member of the infinite Set A*. With this terminology, concept and corresponding limitation in mind, is Set B* a subset of Set A*?

Since the 0.123... element in Set B* has the same representation as the irrational number 0.123... and as n => oo, the difference between the 0.123... element in Set B* and he irrational number 0.123... approaches zero, are these different numbers. If so, in what way do they differ from 0.999... and 1?