Re: Is This System Solvable?

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From: Eugene Shubert (*GalileoProject002_at_everythingimportant.org*)
Date: 02/04/05

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Dave Rusin wrote:
> Something tells me you didn't ask what you wanted to ask.

Thank you Dave. I stand corrected. How about the following restatement:

I'm looking for all sufficiently differentiable real−valued functions of three real variables T(R,S,w) defined everywhere except the point w=0, that have these properties:

For all X, Y, a, b, such that a is not equal to zero, b is not equal to zero, and a+b is not equal to zero, there exists a unique Z=Z(X,Y,a,b) such that the following identities are always true:

T(X, Y, a) = T(X, Z, a+b)

T(Y, X, −a) = T(Y, Z, b)

T(Z, Y, −b) = T(Z, X, −a−b)

Note that the uniqueness of Z = Z(X,Y,a,b) is quite remarkable in that Z is defined by three different equations!

I am also requiring the symmetry that there is a unique X = X(Y,Z,a,b) that satisfies all three functional equations for all Y, Z, a, b, such that a is not equal to zero, b is not equal to zero, and a+b is not equal to zero. Similarly for Y.

I claim that the function T(R,S,w) = R/tanh(w) − S/sinh(w) has all these properties. However, I'm looking for the most general solution to the problem.

As I said before, I vaguely remember something about rank and the Jacobian of a transformation begin zero in certain circumstances, and I assume that a system of PDEs may arise from my three functional equations from that angle.

Any insights would be greatly appreciated.