Re: linear algebra−−can someone check my work

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Date: 02/07/05

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Date: Mon, 07 Feb 2005 08:47:11 +0100
To: tsmith <tsmith76@yahoo.com>
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On 07.02.2005 03:26, tsmith wrote:
> *Q. Consider the case with V being the kth order polynomials with real*
> *coefficients. Let the derivative mapping D be the transformation which*
> *assigns to each polynomial function its derivative. Show that D maps V into*
> *V. What is the rank, nullity, nullspace, and range of D?*
> *==*
> *This is what I did:*
>
> *Let p = a_0 + a_1 x + a_2 x^2 +...+ a_k x^k in V.*
>
> *D(p) = a_1 + 2 a_2 x + ... + k a_k x^(k−1).*
>
> *So D(p) in V since it is a polynomial of at most k.*

Ok.

>
> *Now the thing with the rank and nullity, is there suppose to be a rigorous*
> *way to show these? The only way I know how to find them is by "eyeballing"*
> *the space.*
>
> *I note that only constants and the zero polynomial have zero derivatives,*
> *hence N(T) = { a_0 | a_0 in Reals }.*

since ... (have a look at the equation D(p)= ...)

>
> *And the range R(T) = {p(x) | p(x) = a_1 + 2 a_2 x + k a_k x^(k−1) }*
>
> *Rank(T) = k*
> *Nullity(T) = 1*
> *Dim(V) = k + 1*
>

You could crosscheck this with the dimension formula of linear maps. And
perhaps you can give a reasoning for Dim(V) = k+1?

J.