

Re: Problem with `big oh' estimates in number theory

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From: Angus Rodgers (angus_prune_at_bigfoot.com)

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On Wed, 09 Feb 2005 16:29:51 +0000, Robin Chapman
<rjc@ivorynospatmtower.freemove.co.uk> wrote:

>Angus Rodgers wrote:

>

>> *This isn't just some `obvious' point, which he expects
>> the mathematically mature reader to work out for him/
>> herself.*

>

>I think it is

Oh, well, I've always known I didn't have that `maturity'
thing! (Only age, and lots of it!)

But – taking my life in my hands, I know! – I will continue
to question whether it can possibly be a good idea to use
a notation $O()$ when you have to check it every single time
it's used, in order to determine what the lower bound x_0
happens to be in each case, when the whole point of the
 $O()$ notation – I thought! – was to enable you to **forget**
about the values of M and x_0 when you **don't** need to
know them.

I think it's a pain, and a quite unnecessary one, to boot.

Remember, you have to question each use of the $O()$ notation,
and trace it back through all the intermediate results
being appealed to, checking (perhaps on paper, perhaps in
your head) what value of x_0 **could have** been recorded at
each stage, if only the author had thought/bothered to do
so. And I can see no compensating advantage whatsoever,
except: (1) adherence to tradition; and (2) the fact that
the $O()$ notation certainly **looks** neat, and sticking an
extra subscript on it (for the value of x_0 when needed)
messes it up no end. But that's a price I'd be willing
to pay, for the simple (and surely obvious) gain in sheer

reliability of reasoning. Stick in the value of x_0 , and you can much more easily tell whether something is a valid proof or not. Leave it out, and your program code (so to speak) suddenly acquires all sorts of confusing global references, violating the `black box' integrity of the theorems to which you are **apparently** (but no longer actually) appealing.

If it's OK to drop the value of x_0 even when it **is** needed, why don't we randomly do this with other bits of information?

The value of x_0 is needed – you're not disputing that – so why isn't it in the book? Is the reader required to pencil in all the required values, for future reference? If so, why not just print them there in the first place?

I don't mean to sound so indignant, by the way. I really like Apostol as an author – honestly! But it's irksome to have identified a problem in somebody's writing (and spent hours and hours thinking it through, and double-checking), only to be told that it's not – it's a problem in my reading!

Why should the reader be expected to jump through so many hoops, when: (1) it is not normal mathematical practice to require this (normally you state precisely the results you plan later to use, and throw away only your temporary, intermediate results and notations); and (2) it looks for all the world like a minor **slip** on the part of the author, which was never intended to be turned into a holy **rule** for the poor old reader? (i.e. me – condemned as a poor reader, and now feeling old!)

I think I'll go and lie down.

>> *If that were the case, then filling in the
>> missing steps of the argument would not have required
>> a strengthening of the conclusion of a preceding theorem!*

What he said. :)

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Angus Rodgers
(angus_prune@ eats spam; reply to angusrod@)
Contains mild peril