Re: Why is an integer finite entity?

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In article <cuijrq$2ee$1@news.math.niu.edu>,
Daniel Grubb <grubb@math.niu.edu> wrote:
>
>>Why is the definition of Z the way it is, and not some (perhaps absurd)
>>definition which would include "integers with infinitely many digits".

>Now lets look at those with infinite expansions. For example, let
>x=1000.... with an infinite number of 0's (working base 10, but other
>bases have similar difficulties). What is 10x? Is it the same as x?

Dan, Dan, you're jumping to conclusions here. This is the 21st century! You've got to be more inclusive! More willing to cast aside your tired pre−conceptions of how things ought to be!

Let's start fresh and see if we're really trapped into any kind of corner with your line of reasoning.

So far, this "new−Z" is just a _set_, and it's a perfectly fine set at that. (I might use the set−theorists' notation 10^N because it can be regarded as the set of functions from N = {0, 1, 2, ...} into 10 = {0, 1, ..., 9}.) Well, maybe it's not that either; is, for example, 010000... in this new set? Does _it_ differ from Dan's x ?
I also don't quite see how the ordinary natural numbers are included, either, so maybe this "new−Z" needs to be something like 10^N union N.
Or maybe it's (10 minus 1) x 10^(N minus 1) union N .
(where of course 10 minus 1 does not equal 9 . It's so cool to speak Set Theory!) I guess that's it: these "numbers" are either natural number or infinite strings of digits starting with a non−zero one.

These new integers are pretty cool. There seems to be no reason to think that they can be partitioned into "even" and "odd", for example.
(Is 3434... even or odd?) See how much more liberating the new things are!

Now, Dan, when you speak of "10x", that must mean you imagine this set to have some sort of product structure. Does it? I'm not sure I know how multiplication is defined. In elementary school I learned an algorithm which would let me define products like ...123 x ...456 , but computing 123... x 456... is harder. Hmm, maybe it's just the same as the decimal expansion of 0.123... x 0.456... but with the decimal point and all leading 0's stripped. But that's not a very satisfying definition, and it makes it unclear to me whether the square of 316227766... is 99999... or 100000... Oh dear! Let's not discuss whether those two are equal −− I've got "999" in my killfile!

Well, no matter; I'll just trust that multiplication is defined in some way that matches what I'm about to say, below.

>What is 10x? Is it the same as x?

Seems like it. Is that a problem? You didn't mention whether these "numbers" are supposed to be related to sizes. (I haven't been following this thread at all. Is there an ordering on these things? I don't see how it can be defined! Is 3434... more or less than 4343... ?) Without any reason to think that 10x needs to be "bigger" than x, I'm happy to have 10x = x.

>If not, how do you distinguish the two? If so, then 10x=x, which algebraically
>leads to x=0.

Really? You need "subtraction" to get that conclusion, don't you? What's subtraction? For that matter, what's addition? Is that supposed to be defined too? I don't understand what 10... + 1 is. But maybe addition of two entries of 10^N is well defined, just like multiplication above.

'Course just because I've given definitions of addition and multiplication doesn't mean they work the way we might think. The fact that 10x can equal x is maybe just a fact of life (it works in Z/9Z, after all); in fact I guess we also have 1000... x = x for every x !

But compare
   (5000... + 5000...) + 1000... to 5000... + (5000... + 1000...)
Or
   (3000... + 4000...) * 3000... to (3000... * 3000...) + (4000... * 3000...)
So you've got an addition and a multiplication, all right, but you don't seem to have distributive laws and so on. So if your computations with 10x and x were going to use the fact that 10x = (9+1)x = 9x + 1x = 9x+x, I would claim that there's a problem with the middle step.

Further, you'd have to use some sort of cancellation property, perhaps some variant of the deduction that a + x = b + x => a = b . But that doesn't seem to be a consequence of the definitions since
   9000... + 1000... = 0000... + 1000...
(Oops! "0000..." is not in the set. But 0 is (I think) and maybe even though addition between elements of N and 10^N is undefined in general, perhaps we can agree that x + 0 = 0 + x for all x, whether finite or infinite. But this then does get us to the point where a + x = 0 + x even though a is not zero, so maybe it would be better to leave "0+x" undefined?)

So, see, I don't think there's any contradiction at all, until you
nasty mathematicians start jumping in with a lot of assumptions about
how things are supposed to work. These new integers are just a set,
and there are commutative addition and multiplication operations defined
on certain pairs of these integers, and that's about all we can say.
Don't assume more properties than that, and you've got a dandy
sandbox in which to play.

dave