Re: juggling's combinatorics problem with siteswap

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A RAPTURE OF SITESWAP

thanks to all replies...

i think these ideas are very interesting and have the potential to turn into a new sub−branch of network theory (i.e. graph theory). I have not been involved in the juggling community much, but as far as i know these ideas are unexplored, and there's not much if any literature on them.

Here's some elaborations.

The period number can be given depending on whether the balls are colored. In common usage, the period of a pattern is given as if all balls are of the same color...

so now a pattern can have attributes of: number of balls, periods (at least two types), number of orbits, distinct orbits (a pattern can have n orbits and some of them being the same), and each orbit has a period. And also ambidextrousness and synchronization considerations.

if we assign a symbol to each concept, then we can explore the mathematical relations between them. For example, what patterns are possible with n orbits? Given ambidextrousness and m balls and n colored period, is it possible to have all orbits up to some ceiling number? And can the complete siteswap info be described by some other minimal ways, as perhaps orbits and their periods that more readily shows certain properties? (as if using relations and generators to describe groups.)

And in general, a body of theorems can be developed that tells us the constrains and relations between these properties.

For example, in the current batch of juggling emulation softwares (i'm aware of Juggling Lab), exhaustive patterns are generated by number of balls, and with filters using the concept of (uncolored) period, max

throw, number of jugglers, ground state patterns or excited state patterns... (and few others, some not pertinent in our context such as multiplexing and sync/async) However, no concept of orbit or colored period is considered.

these theories can constitute a systematic study related to directed graph theory, with a concept of a pulse (i.e. the regular "throw") For instance, so far we've considered juggling by a single person. When considering two persons interchanging balls, then the associated notation and pattern variation gets more complex. As common in club juggling, we can consider n person juggling balls. The possible patterns are extremely complex, and spectacular when we consider the questions of orbits. We can consider a hand as a node in a network. When a ball leaves a hand to another hand, it constitutes a directed graph in a network. And the clock−regularity of throwing added to the directed network a concept of tick. (now with respect to this pulsation is the notion of periods.)

these are just some thoughts. I'm in particular interested in the orbit concept. Asides from the math aspect,i like to know the orbits of patterns i'm juggling. (it's not practical to tell when actually juggling as that distracts; and impossible for higher number of balls one can't even hope to juggle.)

one thought started me thinking about these is that i've always been interested in the various mathematical screen savers. (there's the famous but trivial strings, and Game of Life, IFS, fractals, Satori, bouncing billiards ... see this site for a collection of screenshots http://www.jwz.org/xscreensaver/screenshots.html) One particular type is showing objects moving in space, 2D or 3D (or high dimensions visualized thru a projection or slice). The way an object (usually just a point) moves can be described by different types of algorithms, such as by artificial life means (as in boids) or mathematical formula (some parametric formula). Now, it is often spectacular to watch objects fly in 3D in some kind of formation. Simple familiar cases are those jet planes in air shows. For example, in some Olympics opening there are dancers of large number with different colored clothing that dynamically form shapes to be viewed from a distance. Now that's a 2D example. Such can be spectacular in 3D, especially made possible by computer animation. For example, in the early video game The Disc of Tron, after passing a level there's a animation of several discs flying in formation in 3D to form a sphere that grows and shrinks in size.

Now, one of the problems is how to describe such 3D path patterns. Artificial life or parametric formulas each gives rather idiosyncratic and inflexible visual esthetics. AL approach does not form artificially regular patterns and parametric path are too regular and uninteresting. Its formulas hard to work with. But now i thought of the siteswap notation and juggling may be a solution. We consider each hand as a node in a network. Now, the hands need not come in pairs, and can be fixed at various coordinate in 3D. Now a throw from one node to another

can be considered as a directed graph, but now adding a specific path. That is, we specify a space curve in which the ball is to travel from node A to B. (and the different curves connecting nodes can be made to be smooth, as using the math of Bezier curves.) And the ticks in the whole network can be smoothed out. In this way, we can built a system that describes such particle orbit formation. Eventually we can consider the space curves themselves being variable, or the node's coordinates themselves following a given system of particle orbit. (e.g. math question: what happens when such is nested?)

I think this could have many applications. Foremost is entertainment. As a screensaver, it would be beautiful, as there are various needs today when playing mp3 music on the computer. I don't know for sure, but i imagine this systematic orbit formation notation and mathematics (the network theory part) can be applied more seriously in science or engineering.

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for those mathematicians who are not familiar with juggling... here's the context:

we are talking about juggling balls. Say 3 balls. One can have tricks as throwing them behind back, or catching them under the leg etc., but these are not considered here. We only consider throwing balls straight up in front of the body. Now, there are many pattern variations. For example, with 3 ball, one can throw one ball to the right from left hand, then the right hand throws a ball to the left, and repeat. This is the most common pattern. (called cascade) Now, another patter has the left hand keep throwing ball to the right, and the right hand catch and quickly pass to the left hand, so the whole thing cycles in a circular pattern. (this pattern is called "shower"). Now when the balls increase in number, the possible patterns increases. (polynomial time?) About 1990 or so jugglers have developed a notation that describes the pattern variations, called siteswap, which is just a sequence of digits. (see here for detail: http://en.wikipedia.org/wiki/Siteswap)

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