

# Re: juggling's combinatorics problem with siteswap

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here's a more organized account with some corrections.

Some definitions

Some formal definitions, to be refined.

\* siteswap notation is finite sequence of positive integers, as in  $\{5,3,4,4,4\}$  (4 balls tennis pattern) that are constrained by these rules: (to do). each number is single digits, then it can be abbreviated to 53444. And siteswap always mean canonical siteswap ... (to do)

\* length of siteswap is the total numbers in the siteswap. i.e.  $\text{length}[53444]=5$ .

\*  $bs$  = number of balls. Number of balls is the arithmetic mean of the siteswap.

\*  $cp$  = a colored period is the number of throws all colored balls return to their position. For example, for pattern 53444, the  $cp$  is 30. (to do: need math definition here).

\*  $ucp$  = uncolored period. The number of throws all uncolored balls return to their position. In 53444,  $ucp = 10$ . In 3-balls cascade,  $ucp$  is 2. (to do: need math definition here).

\*  $rucp$  = reduced uncolored period. Like  $ucp$ , but mirror cycles are excluded. That is,  $ucp$  is a multiple of  $rucp$ . For example,  $rucp$  for 4-balls tennis 53444 is 5.  $rucp$  for 3-balls cascade is 1. ( $rucp$  can be computed as the total numbers in siteswap sequence.).

\*  $obs$  = the number of orbits in a pattern. (a orbit is the path a ball travels.) In 53444,  $obs = 2$ . (note: each orbit can have its own  $cp$ ,  $ucp$ ,  $rucp$  and siteswap notation.)

\*  $amb$  = true if ambidextrous, otherwise false. A pattern is

ambidextrous if left and right hand play the same role. (if length of siteswap is odd, then ambi is true for sure. 8448 is a counter example)  
... the above needs to be refined and adding more concepts.

### Some Siteswap Examples

- \* single handed 3 balls: 60. obs=1.
- \* single handed 3 balls: 8040. obs=2.
- \* single handed 3 balls: 804060. obs=3.
- \* 3-balls snake: 55050.
- \* 3-balls snake: 7070700.
- \* 3-balls flash 55500. obs=1.
- \* 3-balls flash 7770000. obs=1.
- \* 4 balls tennis: 53444. cp=30, ucp=10, rucp=5, amb=true, orbits={44403,50000}.  $53444=44403+50000 = 3*400040004000300 + 50000$ .
- \* 2 balls shower: 31
- \* 3 balls shower: 51
- \* 4 balls shower: 71
- \* 4 balls cascade: 55550
- \* 5 balls cascade: 55555
- \* 3-balls shower on one hand, 2-balls shower on the other: 64
- \* each hand doing single handed 3-balls 8040: 8844.  $8844=8040+8040$ .

### Some problems to do

- \* Given a siteswap, decompose the siteswap into siteswaps of its orbits.
- \* some orbit's siteswap can be reduced in height. (need to define this more rigorously) For example, 55050 and 7070700 are both 3-balls snakes. Find a way to define the notion of height or equivalence.
- \* investigate composition of siteswaps. That is, given two siteswaps, find another pattern that contains these two as orbits.
- \* decompose a orbit's siteswap into the siteswap of each balls. For example, 3-balls cascade {3}, there is a single orbit that's the siteswap itself. Each ball's siteswap is 300. In 4-balls tennis 53444, one of the ball's siteswap is 400040004000300.
- \* When studying handed juggling of 3-balls, there are 60 (obs=1), 8040 (obs=2), and 804060 (obs=3). Note the 0 there. Perhaps there needs to be a notation with respect to a single node. The siteswap system is based on 2 nodes. One needs to investigate how to extend it to n nodes.
- \* note in 8448 where each hand is doing single handed 3-balls 8040, there's a notion of phase shift.
- \* in pattern 64645 (5 balls), there are 3 orbits, 2 distinct ones. Essentially, each hand independently juggles 2 balls, and another ball goes from hand to hand by itself. Here's a notion where how many nodes

sci.math: Re: juggling's combinatorics problem with siteswap

a ball visits. Given a single ball siteswap, if there are odd number in the sequence, then the ball visit both hands.

I've archived the above at

[http://xahlee.org/Periodic\\_dosage\\_dir/t2/siteswap.html](http://xahlee.org/Periodic_dosage_dir/t2/siteswap.html)

Please post corrections and suggestions. Thanks.

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[http://xahlee.org/PageTwo\\_dir/more.html](http://xahlee.org/PageTwo_dir/more.html)