Re: John Gabriel's Theorem Revisited.

Source: http://sci.tech−archive.net/Archive/sci.math/2005−02/7186.html

---

From: Yan Zhang (*yanzhang_at_fas.harvard.edu*)
Date: 02/18/05

```
Date: Fri, 18 Feb 2005 18:25:15 −0500
```

Jason wrote:

> *Hello Yan,*
>
> *It's simple: w/n can never be zero, no matter how large you make n.*
> *Thus*
> *the denominator in gabriel's quotient may never be zero. Why? Because*

Err. . . but we are taking a limit. Are you saying that
in Lim 1
   x−>3 −−−
         x

the denominator may never be 3? I hope we agree that this gets to the limit 1/3, even though the denominator x "can never be 3, no matter how large I make n".

> *infinity is not a number. In the classic definition, no matter how*
> *small*
> *you make w, it will eventually reach zero. Why? Because zero is a*
> *number.*
> *The difference between zero and infinity is that we can perform finite*
> *calculations/arithmetic using zero but not using infinity. This is the*
> *major difference.*

Um. . . right. . .

I think we not only have a difference of opinion of the derivative, but a difference of what "limit" is. If that is the case, I ask you to give a rigorous definition of "limit" in which Gabriel's derivative or his theorem can be examined. If you cannot supply that then we cannot take this scrutiny further reasonably.

> *The next difference is that gabriel's version leads to the integral*
> *concept whereas the classic version does not. How? Well, when we*
> *integrate, we are summing an infinite number of areas or partitions to*
> *compute the total area/integral. Partitions are part of gabriel's*
> *version but the classic version shows only one partition, i.e w.*

Err. . . surely you are joking? The classic version in real analysis does exactly that, summing an infinite number of areas or partitions using the x−axis to define areas (at least in Riemann− Stiljes − but in Lebesgue integration it does something different, yet of almost same flavor). I do not understand what this "only one partition" thing comes from.

> Now for the most important difference:
> Gabriel's theorem forges the link between the mvt and ftoc to explain
> everything in terms of the average tangent/derivate gradient/value.
> Gabriel's version allows one to pull out the inner limit because both
> are taken to infinity. The classic version cannot be used because the
> limits are calculated differently. This appears to work in gabriel's

. . . I think we definitely have a different idea of what "limits" are and how we can use them. Please enlighten me with yours.

> proof of the average tangent theorem. However, what seems to be suspect
> in his proof is the use of what he calls positional derivatives (*). If
> indeed these positional derivatives exist, then it appears to agree
> with his requirement that the whole interval be differentiable. This
> makes sense because this is exactly what happens when we integrate:
>
> f(x+w) − f(x) 1 n−1 ws 1 x+w
> −−−−−−−−−−−−− = Lim − Sigma f'(x+ −−) = − INTEGRAL
> f'(x)dx
> w n−>Infinity n s=0 n w x
>
> The middle portion in the above is gabriel's ATT (Average of Infinite
> limit
> of the sum of derivatives). It follows that all the interval must be
> differentiable except possibly at x+w.

When you say "it follows", I think you mean, "it is then a necessity that. . ." for this integration.

> 1 n−1 ws
> Average Derivative = Lim − Sigma f'(x+ −−)
> n−>Infinity n s=0 n
>
>
> In the classic format we have:
>
> x+w
> f(x+w) − f(x) = INTEGRAL f'(x)dx
> x
>
> This is how Gabriel explained it to me. Where my wheels come off are
> his positional derivatives are used. They appear to be elusive
> quantities
> since they cannot be calculated but he claims they exist. His proofs

> use
> these positional derivatives.

And we cannot give a serious look at the ATT without looking at these derivatives. Furthermore, we cannot give a serious look at these derivatives until we have the same idea of what a "limit" is. This makes me very uneasy.

[snip]
> So I think I have returned the favour.

To an extent. Your answers are still a little vague, but I'll try my best to keep up. The best way would be for me to actually get some answers on my questions which arose on specific numerical issues.

>>1) We are already depending a lot on real analysis with definition of
>> continuity and limits anyway. I have always been curious exactly
>> which part of real analysis you have problems with, can you point
>> them out again?
>>2) In some sense, we "are" doing real analysis. Though this is just
>> a matter of word choice.
>
>
> No. We are not depending at all on R.A (real analysis) when we use
> gabriel's theorem. This forum is not about the problems I have with R.A
> and there are too many to enumerate here. R.A does not satisfactorily
> provide
> proof for the mvt − this is the main problem. And Rudin's 'proof' does
> it no justice at all.

Exactly, WHICH part is not satisfactory, and WHICH part does it no justice? Please be specific. These attacks on an established mathematical field are unfounded unless you make them pointed and clear, i.e. do some actual mathematical argument.

>>Can you be specific − i.e. mathematically, step by step pointing out
>>this difference? This logic sounds hand−waivy, and I am not competent
>
>
>>enough to understand it in its current form if it is currect. Also,
>>exactly how WHAT is different to WHAT I have stated. I cannot follow
>>this rebuttal. Replies to problems I have posed in the previous post
>>would also be appreciated.
>
>
> I think you are quite able to understand it. Probably more competent
> than any one else in this forum so far. If you stay with it, you may
> even be able to find the answers to my questions before I do. :−)

Again sir, you fail to make EXACT differences, which I really need. Basically, doing some derivatives and show me the difference.

−Yan