

## Re: Contractible metric space

**Source:** <http://sci.tech-archive.net/Archive/sci.math/2005-02/9880.html>

---

**From:** Lasse ([lasse\\_remp\\_e\\_at\\_yahoo.de](mailto:lasse_remp_e_at_yahoo.de))

**Date:** 02/27/05

Date: 27 Feb 2005 11:08:23 -0800

> That's what I had thought, but my problem is that although I can show  $h_x$ :  
>  $[0,1] \rightarrow X$ ,  $h_x(t) = h(x,t)$  is continuous (using this definition of  
> starlike), I cannot show  $h_t: X \rightarrow X$ ,  $h_t(x) = h(x,t)$  is continuous.  
Working  
> in  $\mathbb{R}^2$  for instance, and considering an arc  $(*p)$ , with a point  $x$  on  
this  
> arc, and a neighborhood  $U$  of  $x$ , why couldn't  $U$  intersect (no matter  
how  
> "small"  $U$  is) another arc at point  $y$  so that for some fixed  $t$ , and  
for some  
> fixed  $\epsilon > 0$ ,  $\text{abs}(h_t(y) - h_t(x)) > \epsilon$  (i.e.  $h_t$  is not continuous) ? That's  
what  
> bothers me.

How about the following (it would seem that this can be done in  $\mathbb{R}^2$ ,  
but perhaps it is easier to define on the abstract level):

How about the following: in  $\mathbb{R}^2$ , take the union of the line segment  
from  $(0,0)$  to  $(1,0)$ ; the line segments  $\{1/n\} \times [1,0]$  for all natural  
numbers  $n$ , and a half-circle (or any other curve) from  $(0,0)$  to  $(0,1)$   
lying in the left half plane (i.e.,  $x < 0$ ) apart from these two  
endpoints. Then it would seem that this set is "star-like" by your  
definition, but it is clearly not contractible.

You could assume that your space is compact and this problem would go  
away, but perhaps you don't want to do that.

Hope this helps,

Lasse

---

([remove.for.spam.maths.warwick.ac.uk](mailto:remove.for.spam.maths.warwick.ac.uk))