Re: Convergence of continuous function

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On 1 Mar 2005 07:08:55 −0800, "krill" <krillqill@hotmail.com> wrote:

>When I read Bartle I find the squeeze theorem which said that a
>function is Riemann integrable iff it is a limit of sequences of
>R−integrable function (converges in the sense of Riemann integral).

What does "converges in the sense of Riemann integral" mean?

>However, I find this theorem somewhat trivial, just as one tell me that
>a continuous function is a limit of sequences of continuous functions.

That's true or not, depending on what sort of convergence
you're talking about.

>So I would like to ask:
>
>1. Is it true that a function f is R−integrable iff for every e>0,
>there exists two continuous function g,h:[a,b]−>R, such that
>
>g(x)<=f(x)<=h(x)
>
>and int(h(x)−g(x),[a,b])<e??

Yes, this is easy to see from the characterization in terms of
upper and lower sums.

>2. If a sequence of continuous function {f_n}, where f_n:[a,b]−>R
>converges to f, and f is bounded, is f R−integrable(or equivalently,
>the discontinuity set of f is Lebesgue measure zero?

What sort of convergence do you mean here?

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David C. Ullrich