

sci.math: Euclid never gave a indirect proof of Infinitude of Primes; his proof was direct

# Euclid never gave a indirect proof of Infinitude of Primes; his proof was direct

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The below is an old 1990s post of mine archived in File 106 of my website  
[www.iw.net/~a\\_plutonium](http://www.iw.net/~a_plutonium)

This exposition is not only a correction of one of mathematic's most famous proofs, Euclid's Infinitude of Primes (IP) but also a correction of the history of mathematics as pertains to that proof. And as what Hardy writes in his eloquent book A MATHEMATICIAN'S APOLOGY "no wrinkles", the wrinkles are three major large gashes. As an outline of this exposition, it contains three main ideas and presented herein in that order as three parts.

Part One: The Natural Numbers are not what they presently appear to be of Finite Integers but are rather instead Infinite Integers.

Part Two: Most professors of mathematics were inept in seeing the difference between a Direct and Indirect proof of IP. This ineptitude caused numerous professors of mathematics to publish their version of IP, and 99% wrong and invalid proofs of IP. Most were wrong about the history of this proof, calling it a reductio ad absurdum but never really looking into the matter.

Part Three: Euclid's proof of the Infinitude of Primes was a Direct Proof of that of increasing set cardinality, and not as the often reported news that his proof was Indirect. That is a falsehood, and the historians of Mathematics must correct their error.

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From: Archimedes.Plutonium@dartmouth.edu (Archimedes Plutonium)

Newsgroups: sci.logic,sci.math

Subject: Euclid's IP proof was direct, and not an indirect proof

Date: 13 Jan 1998 07:32:01 GMT

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--- quoting

<http://aleph0.clarku.edu/~djoyce/java/elements/bookIX/propIX20.html>

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EUCLID'S ELEMENTS  
BOOK IX

Proposition 20

Prime numbers are more than any assigned multitude of prime numbers.

Let A, B, and C be the assigned prime numbers.

I say that there are more prime numbers than A, B, and C.

Take the least number DE

measured by A, B, and C. Add the unit DF to DE.

Then EF is either prime or not.

First, let it be prime. Then the

prime numbers A, B, C, and EF

have been found which are more

than A, B, and C.

Next, let EF not be prime. Therefore it is measured by some prime number. Let

it be measured by the prime number G.

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I say that G is not the same with any of the numbers A, B, and C.

If possible, let it be so.

Now A, B, and C measure DE, therefore G also measures DE. But it also measures EF. Therefore G, being a number, measures the remainder, the unit

DF, which is absurd.

Therefore G is not the same with any one of the numbers A, B, and C.

And by

hypothesis it is prime. Therefore the prime numbers A, B, C, and G have been

found which are more than the assigned multitude of A, B, and C.

Therefore, prime numbers are more than any assigned multitude of prime numbers.

Q.E.D.

--- end quoting

<http://aleph0.clarku.edu/~djoyce/java/elements/bookIX/propIX20.html> ---

The language should have helped those in the future to ascribe Euclid's IP as a direct proof, and not the mistaken indirect proof method. By language I mean "assigned multitude" should have warned those judging the proof of Euclid that Euclid gave a direct proof. Assigned Multitude is set theory. And when you want to increase the cardinality of any or every finite set of primes, that is the direct proof of IP.

The word "absurd" was a Roman word, not Greek and I suspect that later writers such as Tartaglia, Cardan and others added the word "absurd".

Word: ab\*surd

Etymology: French absurde , Latin absurdus.

The proof method of Reductio Ad Absurdum is much more than simply

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adding the word absurd somewhere in the text of the proof. Thus, I suspect that the method was not known by Euclid and not by Archimedes either. I suspect the method was not known by humanity until the first official writing elucidating the Method of Reductio Ad Absurdum. Who was that? Perhaps some logician.

And a nice correlating fact is that the ancient Greeks did not have modern set theory, yet they used set theory anyway. And the ancient Greek mathematicians such as Euclid and Archimedes, I contend did not have the method of Reductio Ad Absurdum, and that modern day historians of mathematics falsely ascribe them with this method of proof. What I believe they were doing was proving by geometrical construction, and if the word or idea of "absurdity" cropped up, it was not that they were using the indirect method but merely using the word. And I have traced back the word "absurd" and it is a Roman word not a Greek word.

Where in mathematics history does the full method of Reductio Ad Absurdum appear? I would guess some logician detailed the method. This is where in mathematics history the method of Reductio Ad Absurdum started. Not with Euclid or Archimedes but much more recent.

If we are to give Euclid the credit for Reduce to Absurdity method, then, give the ancient Greeks also the credit for Set theory. But both simultaneously would be a falsehood of math history.

I do not know where in the history of mathematics that the entire math community went astray and not until the 1990s is this able to be corrected.

Perhaps it was a desire of mathematics historians to look for the first recorded indirect proof method, and their desire was more for an earlier and earlier date. Thus, to them, wherever they could get away with it they wanted to ascribe the earliest proof possible for the indirect method. Even though the proof is clearly not the indirect method. And probably none of the Ancient Greeks were consciously aware of the logical mechanisms of the indirect method. I have seen it written some place where Archimedes is ascribed with double reductio ad absurdum methods in a proof. I suspect that is a falsehood and that the authors are seeing and judging those proofs incorrectly. Those ancient proofs of Archimedes were geometrical constructs in the first place. And geometrical construction proofs are direct.

The concept of "assigned multitude" is set theory and thus Euclid's IP was direct proof method.

I doubt that the wording of Euclid's original proof ever had the Reductio Ad Absurdum logic outline of this:

- Suppose a contrary statement,...
- steps of proof
- premissa step
- reach contradiction
- reversal of the supposition statement

Euclid may have had a word like "absurd" or this word may have been added in translated editions of Euclid, and later readers judged from that one word that Euclid gave a indirect proof when in fact he never did. And I argue that the Ancient Greeks, none of them knew or did any mathematics with the Reductio Ad Absurdum. I suspect this method of proof was no older than several centuries when logic was formalized. You need the logical formalization to see how the method of proof delivers a true conclusion. Perhaps the first person who discusses the indirect proof method to any depth was the actual first discoverer of the indirect proof method.

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whole entire Universe is just one big atom where dots of the electron-dot-cloud are galaxies