

Re: Epistemology 201: The Science of Science

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Tony Orlow (aeo6) <aeo6@cornell.edu> writes:

>Neil W Rickert said:

>> *It depends on what you mean by "compare". You can define cardinality
>> using the Schroeder–Bernstein theorem. In that case, you do not make
>> any reference to order.*

>*Can you give an example, say, of comparing the integers and the
>rational using this method?*

This involves defining cardinality in terms of mapping. Set A has the same cardinality as set B if there is a one-to-one mapping of A onto B.

Schroeder Bernstein shows that if there is a one-to-one mapping of A into a subset of B, and if there is a one-to-one mapping of B into a subset of A, then there is a one-to-one mapping of A onto B.

In the case of integers and rationals, we can map the integers into the rationals in the obvious way. That is, the integer n maps to the fraction $n/1$. To map rationals to integers, we proceed as follows: given a positive rational, express as a fraction a/b where a and b are integers, and the fraction is in simplest form (no common divisor of a and b). We map this rational to the integer $(2^a)*(3^b)$. And we map $-a/b$ to the negative of this. We map $0/1$ to 0.

Then Schroeder–Bernstein shows how to construct a one-to-one mapping of the integers onto the rationals.

Sorry if the above is a little technical.