Re: The definite integral.

Source: http://sci.tech−archive.net/Archive/sci.math/2005−03/5498.html

---

From: David C. Ullrich (*ullrich_at_math.okstate.edu*)
Date: 03/16/05

```
Date: Wed, 16 Mar 2005 05:20:03 -0600
```

On 15 Mar 2005 19:13:47 −0800, "William Hughes"
<wpihughes@hotmail.com> wrote:

>
>David C. Ullrich wrote:
>
> <snip>
>
>
>> You've said a few times that one can prove ftc without mvt.
>> How do you do that? Without using mvt at _any_ point in the
>> argument, I mean. (Like, the fact that f' = 0 implies f is
>> constant uses mvt, at least the obvious/standard proof does.)
>
>
>I am pretty sure I have seen an elegant proof of this, but
>I don't have a reference. Perhaps I dreamt it.
>
>However, one can always cheat.
>
>We only need mvt to show f' = 0 implies f constant.
>
>So let f'= 0 on (a,b). Assume wlog that f(0) = 0.
>Assume that f is not constant. Then, there is some
>c in (a,b) with f(c) <> 0. Draw the line from (a,0) to
>(c,f(c)). Note that the slope of this line is not 0.
>Consider the case where the slope is positive.
>Assume that there is a point d in (a,c) with f(x)
>below the line. But now f(x) must cross
>the line from below, or be below the line at c.
>Either leads to a contradiction.
>Assume therfore that f(x) lies above the line.
>By continuity of f, there must be a point d in (a,c)
>such that f(d) < f(c)/2. Draw a new line from
>(d,f(d)) to (c,f(c)). This line also has positive
>slope and f(x) must start out below it (f'(d) = 0).
>But this means that f(x) must cross the second line from
>below, or be below the second line at c. Both

>lead to contradictions.
>The case for negative slope is similar.
>
>Not only is this as ugly as sin,

Yes,

>but it has the same
>"feel" as the usual proof of the mvt.

precisely. I didn't claim that it's necessary to actually _prove_ mvt first − no theorem is ever necessary, you can always avoid using it by inserting the proof instead of citing the theorem.

>Still I can
>claim honour is satisfied.
>
>
> *<snip>*
>
> − *William Hughes*

****

David C. Ullrich