

Re: Nice little total order problem.

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From: William Elliot (*marsh_at_privacy.net*)

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On Thu, 17 Mar 2005, Butch Malahide wrote:

>

> *Stephen J. Herschkorn wrote:*

>> *Butch Malahide wrote:*

>>> *One of them is a set of order type eta times omega-one. That is, the*
>>> *set of all ordered pairs (a,r), where a is a countable ordinal and r*
>>> *is a rational number, ordered by first differences. In other words, a*
>>> *copy of the rational line is inserted between each countable ordinal*
>>> *and its successor.*

>>>

>> *I don't understand. Is this the same as the lexicographic ordering on*
>> *aleph1 x Q?*

>

> *I think so, but I was wrong in thinking that it would have a first*
> *element.*

>

Yes, your 1st construction was exactly that.

No, your 2nd construction is, in order stated, a verbal description of

$$0 + \mathbb{Q} + 1 + \mathbb{Q} + 2 + \dots$$

or the lexicographical order of $\omega_1 \times ([0,1) \wedge \mathbb{Q})$

which I think differs from $\text{lex } \omega_1 \times \mathbb{Q}$ only in having a first element.

>> *If so, how is the interval between (1,0) and (2,0) isomorphic to*

> *Q?*

>

> *That interval contains the pairs (1,x) where x is a positive rational*
> *number, followed by the pairs (2, y) where y is a negative rational*
> *number. That's a countable set, dense in itself, with no first or last*
> *element; so it's isomorphic to Q.*

>

Exactly so, the canonical description of the order type eta.