Re: Distinct linear orderings on Z

Source: http://sci.tech−archive.net/Archive/sci.math/2005−03/7388.html

---

From: Dave Seaman (*dseaman_at_no.such.host*)
Date: 03/21/05

```
Date: Mon, 21 Mar 2005 18:18:41 +0000 (UTC)
```

On Mon, 21 Mar 2005 11:54:55 −0500, Tony Orlow wrote:
> Dave Seaman said:

> Thanks for the understanding and explanation. The "order−isomorphism" you refer
> to sounds much like what I am suggesting. I quickly googled the term and got a
> basic definition, but I wonder if this approach has been applied to deriving
> finite ratios between the sizes of infinite sets, such as between [0,1] and
> [0,2], or finite offsets, such as between (0,1,2...) and (1,2,3,...)? From what
> I read about order types, this extension has not been realized. Is there
> another term I should be looking for?

There are other names for these concepts. There is a branch of analysis called measure theory that considers a generalization of the concepts of length, area, and volume. If we use m to denote Lebesgue measure, then here are some facts about the measure of various sets of real numbers:

(1) m([a,b]) = b − a whenever a < b (the measure of an interval is its length),

(2) m(A) = 0 for every countable set A (such as the set of rationals, the set of integers, or the set of primes),

(3) m(C) = 0, where C = the Cantor set (an uncountable set).

(4) there are sets that are order−isomorphic to the Cantor set but have positive measure.

Another term that is related to things you have said is "density". For example, you used density when comparing the sets (a) multiples of 1000, (b) squares, and (c) primes. All of these sets have measure 0, because they are all countable, but they have different asymptotic densities. This is a concept that depends on the ordering of the real numbers.

So the basic problem is that there may be more than one way to compare sets. Mathematicians avoid confusion by assigning different names to these different concepts. It's possible that sets such as the reals in [1,2] and the rationals in [2,4] may give different results when compared in different ways.

> *It seems to me that the term "number of elements", being constructed from the*
> *fairly simple concepts of "number" and "thing", is a rather general term. The*
> *more exact term as it stands is "cardinality", so it seems to me that if that*
> *is what one is referring to, then that's the word they should use. To equate*
> *cardinality with "number of elements" for infinite sets, to the exclusion of*
> *any other definitions or approaches to defining that vague term, is a mistake*
> *as I see it. It's not that cardinality is a mistake, just that assumed*
> *implication, and I am not at all convinced that it's a semantic or*
> *philosophical matter, when talking about consistent mathematical results.*

But you are overlooking the fact that "number of elements" has no mathematical definition and no one is actually claiming to have proved anything about the meaning of "number of elements" as a phrase in the English language when applied to infinite sets. You are confusing formal mathematical terminology with informal usage.

```
--
Dave Seaman
Judge Yohn's mistakes revealed in Mumia Abu-Jamal ruling.
<http://www.commoncouragepress.com/index.cfm?action=book&bookid=228>
```