Re: JSH: Critique means slow, and thorough

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- *Date*: Fri, 01 Apr 2005 19:02:02 GMT

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jstevh@xxxxxxx wrote:

> Jesse F. Hughes wrote:
>
> ... stuff deleted ...
>
>> You may ask how we know that there *exists* a least ring containing both Z and 1/2, but that's a different (and easy) matter. But the fact is, if Z[1/2] denotes any ring at all, then it denotes a ring contained in Q.
>>
>> Honest, it does.
>
> Well, it'd be nice if it did, but it doesn't.
>
> AND HERE'S THE PROOF:
>
> Think of coprimeness as being like a filled balloon.
>
> If you prick the balloon by introducing something that contradicts any coprimeness result in the ring of integers, like sticking in 1/2 so that 2 is no longer coprime to 1, then you pop the balloon.

Now, *that's* some high−falutin' math, there!

Here's some more:

> Think of your balloon as a flaming sack placed
> on your doorstep just before someone rings the
> doorbell.
>
> Think of the partial sums of a convergent series
> as your foot, instinctively coming down to put
> out the flames.
>
> Think of the convergence of this series as your
> foot coming down hard on this flaming sack.
>
> Think of how much of your own concept of mathematics
> you're going to have to scrape off your shoe.

Why is your model of coprimeness any better than my model of
your model of mathematics?

> Now you can *define* a minimal ring all you want, except that the ring
> is infinite in size. Because the ring is infinite in size, what you're
> doing is like trying to look at only the foot of an elephant and
> *define* that foot to be the entire elephant.

But an you can't fit an elephant, not even an elephant's foot, inside
a filled balloon. The balloon is FULL! Everyone knows that FULL means
that nothing else can get into that thing!

> Mathematically the *definition* has no meaning, and you get infinite
> sums whether you want them or not.

> One way to look at it is that to the mathematics either you have coprimeness rules or you don't.

Hmm. So, what you're saying, if I get the meaning correctly, is that there is a single "coprimeness" concept, and it doesn't really depend on what elements in the ring you're talking about, or even about the ring itself. It's just "coprimeness".

> If 1 and −1 are the only rationals units in the ring, like with integers, plus my other requirement, then you have a ring where coprimeness holds.

Your "other requirement" ? What "other requirement" would that be?

> If you break the coprimeness rules even a little bit, you break them completely.
>
> You cannot break them halfway.
>
> The idea that you can just stick in 1/2 and get a partial break is just wrong.
>
> It's a complete break.
>
> It even sounds bogus when you think about it that arbitrarily you can remove convergent sums just because they're infinite sums.
>
> Hey, they converge, they're in there.

So, you're saying that the ring of algebraic integers is really the full field of complex numbers, right? After all, given *any* complex number z, there is a sequence (well, many sequences, to be sure, but at least one) of algebraic integers that converges to z. By taking successive differences of this sequence, we obtain an infinite sum of algebraic integers that converges to z.

Explicitly: let z be any complex number, and $\{a_n \mid n \geq 0\}$ a sequence of algebraic integers that converges to z.

For $n \geq 1$, let $b_n = a_n - a_{(n-1)}$. Then b_n is an algebraic integer for each $n \geq 1$.

Consider the infinite sum:

$$a_0 + b_1 + b_2 + \ldots + b_n + \ldots$$

It is trivial to verify that for $n \geq 1$ the nth partial sum is equal to a_n:

$$S_n = a_0 + b_1 + \ldots + b_n$$

$$= a_0 + (a_1 - a_0) + (a_2 - a_1) + \ldots + (a_n - a_{(n-1)})$$

$$= a_0 + a_1 - a_0 + a_2 - a_1 + \ldots + a_n - a_{(n-1)}.$$

$$= a_0 - a_0 + a_1 - a_1 + a_2 - a_2 + \ldots$$

$$+ a_{(n-1)} - a_{(n-1)} + a_n$$

$$= a_n.$$

Since the sequence {a_n | n >= 0} converges to the (arbitrarily chosen) complex number z, by definition

z = a_0 + sum(b_n, n=1,..., infinity).

According to your reckoning, then, z is in the ring of algebraic integers.

For instance, 1/2 is an algebraic integer.

> So, mathematically, despite what semantics you use, when you break the integers, you get reals, whether you want to or not.

Here's the statement you want to assert as a theorem:

CLAIM (JSH): If K is a field, and sum(k_n | n >= 0) is a convergent infinite series of elements of K, then the limit

S = lim_{n --> infinity) (k_0 + ... + k_n)

is an element of K.

Here's the corollary that you are then obliged to acccept:

COROLLARY: pi is a rational number.

PROOF:

1. Q, the set of rational numbers, satisfies the field axioms, and is therefore a field.

2. The finite set of decimal expansions of pi form the sequence of partial sums for a particular series of rational numbers.

3. The series of statement 2. converges to pi.

4. The summation of that series, defined as the limit of the sequence of partial sums, is equal to pi.

5. pi is therefore an element of Q.

If the above CLAIM(JSH) is assumed, then the above is a correct proof, modulo details about defining convergence (which I take it you assume).

The justification for each step (each may take a certain amount of work to verify, but each step is simple to do):

1. Definition of field, definition of rational numbers.
2. Take the series formed by adding the sequence of differences.
3. Convergence in R is guaranteed by the fact that the sequence of finite initial segments of the decimal expansion of any real number is a Cauchy sequence.
4. Definition of the sum of an infinite series.
5. CLAIM(JSH).

As I said, given your claim, you must then accept the rationality of pi. By a similar token, with a different convergent series, you must accept that pi is an algebraic integer, leading to the following

ASTONISHING CLAIM (JSH)

pi is an integer!

PROOF:

pi is a rational number
pi is an algebraic integer

The intersection of the algebraic integers
with the rational numbers is equal to the
set of (ordinary, rational) integers.

==> pi is an element of Z.

So, not only must you agree that pi is rational, you must stand with the Great State of Indiana in insisting that pi is an integer!

WOW!

James Harris

This is fun. Just how much trouble can a person get into by holding an apparently innocent position, and disregarding the constraints imposed by standard definitions?

Dale
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