Re: Coprimeness − I think I'm confused, but I'm not sure

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- *From*: David Rosoff <rosoff@xxxxxxxxxxxxxxxxxxxx>
- *Date*: Sat, 2 Apr 2005 13:22:49 −0800

On Fri, 1 Apr 2005, Matt Gutting wrote:

> Arturo Magidin wrote:
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> > In article <1112387619.8214cb7fca92c72ca9857edb4554270a@teranews>,
> > Matt Gutting <matthewdba@xxxxxxxxxxx> wrote:
> >
> > > In light of some of the comments in the JSH threads involving coprimeness and Z[1/2]:
> > >
> > > If the definition of coprimeness is something like:
> > >
> > > To say that 'p and q are coprime in a ring' is to say
> > > 'there exist a,b in the ring with ap + bq = 1'
> >
> > This is true in rings with 1.
>
> Okey dokey. I guess it's difficult for me to work with rings
> lacking a unit element because the books I've used (Herstein,
> and Rotman), while mentioning that "not all mathematicians
> insist that a ring contain a unit element", do in fact insist
> for the purposes of the book that rings be required to have
> a unit element.

> You wouldn't know of a (beginning grad−level) book that discussed
> the differences between rings with and without unit element?

Check out Harry C. Hutchins' book, "Examples of Commutative Rings." There is a shortish section about rings without 1. I agree with Dale Hall's sentiment, expressed in one of the JSH threads, that not having a 1 makes ideals turn stupid. For example, in such a ring, maximal ideals need not be prime. Let R = 2Z, M = 4Z. Then M is a maximal ideal, but it is not prime (2*2 is in M).

The book is also quite good for one who wants more familiarity with the menagerie of commutative rings, especially the differences between Noetherian and non−Noetherian rings. I particularly liked the example of a ring R of Krull dimension 1 such that R[x] has Krull dimension 3. Unfortunately I don't remember the construction.

HTH,

−dave

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