

Re: how to deal with the algebra problem?

Source: <http://sci.tech-archive.net/Archive/sci.math/2005-04/msg02224.html>

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- *Date:* 18 Apr 2005 07:21:59 GMT

In article <A11622TF\$scimath@xxxxxxxxxxxxxxxxxxxxxxxx>, SU(2) <idyllic.bbs@xxxxxxxxxxxxxxxxxxxxxxxx> wrote:

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>assume a1*a2*...*an=1
>does there exist b1,b2,...,bn such that:
>a1(b1/b2)=1
>a2(b2/b3)=1
>.
>.
>.
>an(bn/b1)=1

```

In addition to the responses you've already gotten (which are the ones you probably wanted) you need to know that you're encountering cohomology of groups here!

Your a's and b's all lie in the group R^* , the multiplicative group of the nonzero real numbers. THE group $M = (R^*)^n$ is also an abelian group but in addition is a module for the cyclic group $G = \mathbb{Z}/n\mathbb{Z}$ of order n , the generator g acting via

$$g((a_1, a_2, \dots, a_n)) = (a_2, a_3, \dots, a_n, a_1)$$

[I don't really care which generator g of G acts this way; if it bothers you, take $g = 1 + n\mathbb{Z}$.]

Next fix G as above but, for the moment, let M be any G -module, and consider the sequence of G -modules and G -module homomorphisms

$$\dots \rightarrow M \xrightarrow{h_1} M \xrightarrow{h_2} M \xrightarrow{h_1} M \rightarrow \dots$$

where $h_1 = 1 + g + g^2 + \dots + g^{n-1}$ and $h_2 = 1 - g$. These bits of arithmetic are carried out in the endomorphism ring of M ; the use of the symbols "+" and "-" is traditional for abelian groups but in your case M is written multiplicatively so these would mean, respectively, $h_1(m) = m \cdot g(m) \cdot g(g(m)) \cdot \dots \cdot g^{n-1}(m)$ and $h_2(m) = m \cdot \{g(m)\}^{-1}$. You can calculate -- without knowing what endomorphism g is, in fact -- that $h_1 \circ h_2 = 0$ and $h_2 \circ h_1 = 0$, which means that the kernel of h_1 contains the image of h_2 and vice-versa. It therefore makes sense to define the cohomology groups

$$H^1(G, M) = \ker(h_1) / \text{im}(h_2)$$

$$H^2(G, M) = \ker(h_2) / \text{im}(h_1)$$

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These gadgets show up all over the place in mathematics, e.g. in Hilbert's "Theorem 90" (in number theory) and in many parts of algebraic topology.

If you do the calculations in your case, you find that

$$h_1(a_1, a_2, \dots, a_n) = (A, A, A, \dots, A)$$

where $A = a_1 a_2 a_3 \dots$; also

$$h_2(a_1, a_2, \dots, a_n) = (a_1/a_2, a_2/a_3, \dots, a_n/a_1).$$

Thus, your question asks precisely, "Is something which lies in the kernel of h_1 necessarily [the inverse of something] in the image of h_2 ?"

That's exactly the question, "Is $H^1(G, M) = 0$?", and the answer (as noted in the other responses in this thread) is "Yes."

I'm almost embarrassed to admit that this is really the way I think about the original question.

dave

• **References:**

◆ **how to deal with the algebra problem?**

◇ From: SU(2)

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