Re: Idiosyncratic (or idiotic?) notation for lim inf, etc.

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- *From*: David C. Ullrich <ullrich@xxxxxxxxxxxxxxx>

- *Date*: Sun, 17 Apr 2005 14:19:36 −0500

On Sun, 17 Apr 2005 19:06:30 +0100, Angus Rodgers
<angus_prune@xxxxxxxxxxx> wrote:

>Having blurted out in passing in another thread that I was
>struggling to understand how to interrelate several forms
>of Fatou's Lemma (and other corollaries of the Monotone
>Convergence Theorem), in the context of the particular way
>that the theory of Lebesgue (or rather Daniell) integration
>is developed in the book [Weir] − when other versions of the
>same theorems are given in the context of several different
>ways of developing the theory − I /think/ I've now got it
>sorted out in my mind, and I /think/ it might make sense
>to post the results here, in a series of short articles.
>
>That may already be a bad idea, for all I know − does anyone
>else but JSH use sci.math to `think aloud'?

I don't see why it's a _bad_ idea, but I'm not sure I see
the point to it, unless you've come up with new and
different proofs. The standard proofs are not all
that hard, and they're easy to find in lots of books.

> − but before I
>even get started on it, I want to check whether it might be
>an even worse idea to use in public the notation that I have
>been using in my own notes.

That strikes me as a bad idea.

>All the results concern a sequence (f_n) of integrable functions.
>In [Weir], all sequences have the index set {1, 2, ...}.
>
>A sequence (s_n | n >= 1) of real numbers (i.e. a real−valued
>function on {1, 2, ...}) I will denote simply by s, and I will
>use `Sup s' and `Inf s' to denote the upper and lower sequences
>determined by s, thus:
>
> (Sup s)_n = sup{s_n, s_{n + 1}, ...} (n >= 1)

> (Inf s)_n = inf{s_n, s_{n + 1}, ...} (n >= 1)
>
>Then Sup s is a decreasing sequence, and Inf s an increasing
>sequence, so the (possibly infinite) limits lim (Sup s) and
>lim (Inf s) always exist, and are the numbers conventionally
>denoted by lim sup_{n −> oo} s_n and lim inf_{n −> oo} s_n,
>so we can denote these numbers briefly, unambiguously, and
>(I hope) not too confusingly by `lim Sup s' and `lim Inf s'.
>Their possible values, in the extended real number system,
>include +oo and −oo. (Then the sequences /diverge/ in R.)
>
>(My notation is probably not suitable for general use, because
>of the risk of confusion with the possible use of sup s, inf s
>to mean respectively sup{s_1, s_2, ...}, inf{s_1, s_2, ...}.
>But I think it causes no confusion in the present context;
>and even more generally, its use of capitalisation to avoid
>confusion with sup and inf might make it just about usable.)
>
>Similarly, rather than use `f' for a limit function (if such
>exists), I'll use it as a brief notation for the /sequence/
>(f_n | n = 1, 2, ...); for any real number x, I'll use `f(x)'
>to denote the sequence (f_n(x) | n = 1, 2, ...); and finally,
>I'll use `Sup f' and `Inf f' to denote the upper and lower
>sequences determined by f. Thus, for all x in R:
>
> (Sup f)(x) = Sup (f(x))
> (Inf f)(x) = Inf (f(x))
>
> (Sup f)_n(x) = sup{f_n(x), f_{n + 1}(x), ...} (n >= 1)
> (Inf f)_n(x) = inf{f_n(x), f_{n + 1}(x), ...} (n >= 1)
>
>Then Sup f is a decreasing sequence, and Inf f an increasing
>sequence, so the limit functions lim (Sup f) and lim (Inf f)
>always exist (but may take infinite values, for some values
>or even for all values of their arguments), and are the same
>functions conventionally denoted by lim sup_{n −> oo} f_n
>and lim inf_{n −> oo} f_n, so we can denote these functions
>briefly by `lim Sup f' and `lim Inf f'. (I'll return later
>to the question of convergence in the /non/−extended set of
>real numbers, R. Here I'll just mention that in [Weir], no
>use is made of the extended real number system; and I don't
>know if other books define the integral in such a way that
>its value can sometimes be +oo or −oo. Perhaps there's
>some `obvious' reason why this can't be done?

No, it's very standard to say that every non−negative
measurable function has an integral, possibly infinite;
if f >= 0 then one often just defines the integral to
be sup int g, for simple functions g with 0 <= g <= f.

>Anyway, I'll

>stick as closely as possible to [Weir], because that's the
>book that I have to read for my course.)
>
>I'll also use `I' as the notation for the integral operator,
>and `If' to denote the sequence of integrals If_n determined
>by a sequence f of integrable functions, thus:
>
> (If)_n = If_n = I(f_n) (n >= 1)
>
>We thus also have two monotone sequences Sup If and Inf If,
>tending always to the (possibly infinite) limits lim Sup If
>and lim Inf If.
>
>(If you don't like these notations, it is at least always
>possible, I hope, to translate them unambiguously into the
>more prolix conventional symbols.)

Uh, this is the end of the post and you haven't made any
mathematical assertions yet? It's really not clear what
the point to this is. Oh, I see another thread where you
do nothing but state the Monotone Convergence Theorem.

I think that maybe if you're going to do this for whatever
reason then at least making it all one thread would be
better. I mean the point to this thread is to set up the
notation you intend to use in other threads, so those
other threads are going to be assuming that the reader
has read all your previous posts, in various different
threads. Seems presumptuous or something.

>Reference
>−−−−−−−−−
>
>[Weir] Alan J. Weir, "Lebesgue Integration and Measure",
>Cambridge University Press (1973).
>
>I also consulted books by J. C. Burkill, J. F. C. Kingman
>& S. J. Taylor, E. J. McShane & T. A. Botts, F. Riesz &
>B. Sz.−Nagy, and A. E. Taylor. Between them, these gave
>me some idea of how to go about organising the material
>which then had to be fitted into the development of [Weir].
>(I didn't bother looking at proofs, which would only have
>confused me.) :)

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- ***Follow−Ups***:
  - ♦ ***Re: Idiosyncratic (or idiotic?) notation for lim inf, etc.***
    - ◊ *From:* Angus Rodgers

- ***References***:
  - ♦ ***Idiosyncratic (or idiotic?) notation for lim inf, etc.***
    - ◊ *From:* Angus Rodgers

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