Re: Convergence of random variables

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- *From*: "Stephen J. Herschkorn" <sjherschko@xxxxxxxxxxxx>
- *Date*: Tue, 19 Apr 2005 18:14:01 −0400

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quantalfred wrote:

> > quantalfred wrote:
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> > > When talking about almost sure
> > > convergence, do we
> >
> > have to clarify the underlying sample space
> > beforehand?
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> > > Say for example, let X_n be iid such that
> > > P(X_n)=1=P(X_n)=0=1/2,
> > > then what's lim X_n?
> >
> > Declaring the randomv variables to be independent
> > implies they are defined on the same probability
> > space.
>
> Then what could you say about this example: ([0,1], B[0,1], Lebesgue measure), set X_n(x) =
>
> What's wrong?

With this explicit definition, the X_i are no longer independent, in contrast to your original statement.

In this example, the sequence diverges almost surely.

Why? By Borel-Cantelli? But isn't it counter-intuitive? All random variables are identi

Cite souces of assistance in submitted homework.

Unfortunately, it's not a homework. I couldn't find any books which deals with the stuff very clearly.

Ah, I thought you were taking a class. Why do you find it counterintuitive? You are flipping a fair coin over and over; the sequence will surely oscillate.

Are you confusing almost sure convergence with convergence in *distribution*? Convergence in distribution is also called weak convergence; do not confuse it with convergence in measure (also called convergence in probability). It is really a statement about the sequence of distributions rather than the random variables. Convergence in distribution is the type of convergence that appears the Central Limit Theorem. It is trivial that your X_n converges to X1 in distribution.

As to showing that X_n does not converge to X_1 almost surely, use the fact that almost sure convergence implies convergence in probability.

References:
Billingsley, Probability and Measure.
Drake, Fundamentals of Applied Probablity Theory.

Ross, Stochastic Processes.

By the way, you do not need an infinite−product space to define a sequence of i.i.d. r.v.'s. Let U be the identity map on [0,1] under Lebesgue measure. Let X_n = [2^n U] mod 2 (i.e., the nth digit in the binary expansion of a single uniform random variable), where [.] indicates the integer floor.

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